Manhattan Mathematical Olympiad 2012

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– Grades 5-6

– **p1.** How many five-digit numbers are there which will produce the number 2012 if one digit is crossed out?

p2. A classroom floor is colored using two colors. Prove that there are two identically colored points exactly 1 foot apart.

p3. Can you draw a closed
a) 6-segment
b) 7-segment
c) 8-segment
polygonal line which self-crosses each of its segments exactly once? Example on the picture shows a 5-segment closed polygonal line which self-crosses each of its segments twice.
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`png`

p4. Jack wants to enter a wonder cave. In front of the entrance there is a round table with 4 identical hats lying symmetrically along the circle. There are 4 identical coins, one under each hat. Jack can lift any two hats, examine the two coins, turn them as he likes and put the hats back. After this Jack closes his eyes and the table starts spinning, and when it stops Jack cannot tell by how much the table rotated. Then again he can choose two hats and so on. The wonder cave will open if and only if the coins are either all HEADS or all TAILS up. How must Jack act to enter the cave?

Remark: Lifting hats randomly and turning all coins, say, HEADS up is not a winning strategy. Jack may be so unlucky that he never lifts a hat which covers TAIL.

PS. You should use hide for answers.

– Grades 7-8

– **p1.** Is the last non-zero digit of the number $1 \cdot 2 \cdot 3 \cdot ... \cdot 2011 \cdot 2012$ even or odd? Justify your answer.

p2. Seven coins lie along a circle HEADS up. Can you make them TAILS up if you are only allowed to turn over any
a) five successive coins
b) four successive coins
at a time?

p3. In a geometry lesson Jay and Bob drew geometrical tadpoles, outlined in bold on the right. (The four circles in the figure have the same radius, the triangle is equilateral, and its horizontal side is a diameter of the circle.) Which of the two tadpoles has greater area?
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p4. Three pawns are placed on vertices of a pentagon. Your goal is to have one pawn in its original position and have the other two switch their places. If you are allowed to move a pawn along sides of the pentagon to any free vertex, then the goal is clearly impossible. Can you achieve the goal if it is allowed to move pawns along diagonals of the pentagon instead of its sides?

PS. You should use hide for answers.

– Grades 9-12

– **p1.** The product of 2012 positive integers is equal to 2012. What is the greatest value the sum of these numbers can have?

p2. Each cell in a 5×41 rectangular grid is colored either red or blue. Prove that some 3 rows and some 3 columns must intersect in 9 cells of the same color.

p3. Show that any convex polygon of area 1 is contained in a rectangle of area 2. (Recall: "convex" means that with any two points the polygon contains the entire segment joining them).

p4. There are two identical jars and a 100 story building. Your goal is to find out the lowest floor such that, being dropped from that floor, jars break. (That is, you need to find n such that the jar breaks if dropped from n-th or higher floor, and does not break if dropped from $n-1$-st or lower floor. Of course, if a jar breaks, you cannot reuse it, but you still can use the other jar). Give a strategy which guarantees achieving the goal in
a) no more than 19 trials;
b) no more than 14 trials.