

Manhattan Mathematical Olympiad 2014

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– Grades 5-6

– **p1.** A dandelion blooms in two stages: first it becomes yellow for three days, and then it turns white for two more days and then it stops blooming. On Monday there were 20 yellow and 14 white dandelions on Jeremy's front yard. On Wednesday there are 15 yellow and 11 white. How many white dandelions will be on Jeremy's front yard on the following Saturday?

p2. There are several kids in a room. If Ann gives half of her candies to Victor, then everyone will have the same amount of candies. If Ann gives all her candies to John, then John will have as many candies as all other kids combined. How many kids are there in the room?

p3. Draw a figure which can both be cut into 4 figures of the shape on the left and in 5 figures of the shape on the right.

<https://cdn.artofproblemsolving.com/attachments/a/c/f2ff4685ecf0b3583b62af00831db7388d941.png>

p4. Ann and Peter are playing the following game. There are two numbers written on the blackboard in the beginning: 2014 and 2015. The players take turns and on each step one of the following operations can be performed:

- (1) The player can subtract one of the digits written on the blackboard from one of the numbers.
- (2) The player can divide one of the numbers by two, given that it is even.

The first player to get a number less than 10 wins. Ann goes first. Which player has a winning strategy?

PS. You should use hide for answers.

– Grades 7-8

– **p1.** In a 5×5 square 25 integer numbers are written such that the product of numbers in every row is negative. Prove that there is at least one column with product of its numbers also negative.

p2. 13 kids sat around a round table and decided that boys will always lie to girls and tell truth to each other, and girls will always lie to boys and say truth to each other. One of them told to the right neighbor: "Most of us are boys". Then the neighbor told to his (or her) right neighbor

"Most of us are girls". And so they went on, alternating the phrases, till the last kid told the first one "Most of us are boys". How many boys are at the table?

p3. The right triangles $\triangle ABC$ and $\triangle DAF$ on the picture below are congruent with $\angle BAC = \angle ADF = 90^\circ$. Prove that the triangle $\triangle ADB$ in the intersection is equilateral.

<https://cdn.artofproblemsolving.com/attachments/6/a/4f3e90d3c066b733da845531b5d5c90041ecc.png>

p4. Among 9 coins: 5 quarters, 3 dimes and a nickel there is one fake coin, it weighs less than it should. By using balance scale (no extra weights allowed) can you determine in two rounds which coin is fake?

PS. You should use hide for answers.

- Grades 9-12

- **p1.** Three real numbers a, b, c are such that $ac + bc + c^2 < 0$. Prove that $b^2 > 4ac$.

p2. Prove that the number $n^4 + 4^n$ is prime if and only if $n = 1$ (here n is an integer).

p3. You are given three equal coins and a pen. A circle with the diameter equal to the one of coins is drawn on a piece of paper. Can you find its center using the three coins and the pen? You are allowed to: • Place a coin passing through two marked points (if possible) • Place a coin touching one or two (if possible) coins • Mark all points on the circumference of a placed coin • Mark the point of tangency of two coins (if they are touching)

p4. Given a triangle in one step you are allowed to change one of its side (so that the result is still a triangle). What is the minimal number of steps you need to transform the equilateral triangle with side 1000 to the equilateral triangle with side 1?

PS. You should use hide for answers.
