Manhattan Mathematical Olympiad 2015

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– Grades 5-6

– **p1.** Is it possible to represent the number 203 as a sum of several whole numbers such that their product is also equal to 203?

p2. A monkey is happy if it eats 3 types of fruit. There are 20 pears, 30 bananas, 40 peaches and 50 oranges. How do you distribute the fruit to make the most number of monkeys happy? What is this number?

p3. Cut the figure below in two identical figures.
https://cdn.artofproblemsolving.com/attachments/9/3/5922398b5fb319994e11033b52118c12a64ab
png

p4. There are 4 intersecting straight line paths in a garden, as shown in the pictures below. Place 4 trees so that there are equal number of trees (two) on both sides of EVERY path! (Practice and put your answer on the LAST picture.)
https://cdn.artofproblemsolving.com/attachments/1/0/fa357c8ac0ffd8d69dba9d3d9b2116044b376
png

PS. You should use hide for answers.

– Grades 7-8

– **p1.** A flashlight produces the light beam which covers a sector of 90 degrees. Show that, given four points in the plane, it is possible to place a flashlight at each one of them, so that every point in the infinite plane is lit by at least one flashlight.

p2. Suppose a point P in the plane is given, together with some some figure F, of which we only know the following: if we rotate F by 48 degrees around P, then the resulting figure is same as F. Can we claim the same property of the figure F for rotations around the point P by:
(a) 72 degrees?
(b) 90 degrees?

p3. Compute $\frac{4^6\cdot 9^5+6^9\cdot 120}{8^4\cdot 3^{12}-6\cdot 11}$.

p4. Is it possible to cut the infinite plane into congruent pentagons, each of which does not have any parallel sides?

PS. You should use hide for answers.

– Grades 9-12

– **p1.** Somebody placed 2015 points in the plane in such a way that any triangle with vertices at those points has the area less or equal than 1. Prove that all points can be placed inside of a triangle which has area equal to 4.

p2. Is the following statement true? *For any point inside of a convex quadrilateral the sum of distances from that point to the vertices of the quadrilateral is smaller than the perimeter of the quadrilateral*.

p3. Prove that the following inequality is satisfied for any $a \neq 0$:

$$1+\frac{1}{a^2} \geq \frac{2}{a}-\frac{11}{25a^2}+\frac{2}{5a}.$$

p4. A sequence of 128 numbers $x_1, x_2, ..., x_{128}$ is given, such that each number is either $+1$ or -1. We construct a new sequence of 128 numbers consisting of products:

$$x_1x_2, x_2x_3, x_3x_4, ..., x_{127}x_{128}, x_{128}x_1.$$

Then we construct a new sequence from this one, using the same rule. Prove that if we repeat the construction sufficiently many times we arrive at the sequence consisting only of $+1$'s.

PS. You should use hide for answers.