Manhattan Mathematical Olympiad 2017

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– Grades 5-6

– **p1.** Three drawers, one with spoons, another with forks and the third one with a mixture of spoons and forks, are labeled "SPOONS", "FORKS" and "SPOONS AND FORKS". Its is known that all labels are incorrect. Can you tell what each drawer contains by just picking one item from one of the drawers (of your choice)?

p2. Four distinct digits are written on two two-sided cards, one digit on each side of each card. Is it possible that all possible combinations of two-digit numbers that can be shown with these cards are prime?

p3. A $2 \times 2 \times 2$ cube is assembled of 8 cubes. The sides of smaller cubes are painted blue or red. One-third of all sides are painted blue and two-third are red. In the assembled cube two-third of the visible sides are blue and one-third are red. Show that you can reassemble the $2 \times 2 \times 2$ cube so that all visible sides are red.

p4. The number 60 is written on a blackboard. Alice and Bob take turns subtracting from the number on the blackboard any of its divisors (including 1 or the number itself), and replacing the original number with the result of this subtraction. The player who writes the number 0 loses. Alice starts. Show that she can always win no matter what Bob's moves are.

PS. You should use hide for answers.

– Grades 7-8

– **p1.** A 3×3 square is filled with numbers $1, 2$, or 3, one number in each of the 9 boxes (numbers can repeat). Can the eight sums along the square's rows, columns, and diagonals be all different? (There are three rows, three columns, and two diagonals, eight in total.)

p2. A right triangle $\triangle ABC$ has right angle at C. Let M and N be the midpoints of the sides AC and BC respectively. Suppose $AN = 19$ and $BM = 22$. Find AB.
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`png`

p3. Four real numbers are written on the blackboard. At each step Kate erases any two numbers and replaces them with a new number which can be either the sum, the difference, the product, or the ratio of the two erased numbers. (Difference and ratio can be taken in either order.) After three steps can Kate get 24, if the original four numbers were
(a) $2, 2, 5, 10$?
(b) $2, 5, 5, 10$?

p4. A group of tourists is having a snack. If they open two identical packages of cookies and share the cookies evenly, there will be 1 cookie left. If they open three packages, there will be 13 cookies left. How many tourists are in the group? Explain why this is the only possible answer.

PS. You should use hide for answers.

– Grades 9-12

– **p1.** It takes a freight train one minute to go past a person standing next to the track, and two and a half minutes to go through a bridge that is 3 km long. How fast is the train going?

p2. Find five non-zero real numbers such that when each of the numbers is decreased by one their product is unchanged.

p3. Is it possible to cut a square into several triangles so that each of these triangles has three acute angles?

p4. Among 18 identical balls two are radioactive. You can test any group of balls for radioactivity: the group tests positive if it contains a radioactive ball. Can you figure out which two balls are radioactive with
(a) 10 tests?
(b) 8 tests?

PS. You should use hide for answers.