Manhattan Mathematical Olympiad 2019

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– Grades 5-6

– **p1.** A teacher is setting up a demonstration before a class of 33 students. She tells the students to close their eyes and not to look. Every boy in the class closes his left eye; one-third of the boys also close the right eye, but two-thirds of the boys keep the right eye open. Every girl in the class closes her right eye; one-third of the girls also close the left eye, but two-thirds of the girls keep the left eye open. How many students are watching the teacher despite her asking them not to?

p2. The numbers from 1 to 100 are written on a 10×10 board in the consecutive order. That is the first row is 1, 2,... , 10, the second row is 11, 12,... , 20 etc. You place 10 rooks on the board so that no two of them are in the same vertical or horizontal row, and add up the numbers under the rooks. What is the largest sum you can get?

p3. Cut the shape below into four equal pieces.
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`png`

p4. A landscaping company planted 2019 trees in a circle around the K-State campus. 1010 of them are oaks and 1009 are maples. Prove that somewhere in this circle there are two oaks with exactly two other trees between them.

PS. You should use hide for answers.

– Grades 7-8

– **p1.** Arrange 9 fruits whose weights are $2, 3, ..., 9, 10$ lbs into 3 light suitcases, 3 in each, such that each suitcase weighs less than 20 lbs and no suitcase contains a pair of fruits whose weights divide one another.

p2. Find all integers n such that $3^n + 4^n = 5^n$.

p3. Build the regular 12-gon with side length 1 using squares with side length 1 and equilateral triangles with side length 1. You can stick the figures together along their sides, but no overlaps.

p4. There are 14 identical true coins and 2 identical fake ones. It is not known if the fake coins are heavier or lighter than the true ones. Using an ordinary balance scale can you divide the 16 given coins into two piles of the same weight in just 3 weighings?

PS. You should use hide for answers.

– Grades 9-12

– **p1.** Does there exist an integer n such that $\underbrace{11...1}_{n}$ is divisible by $\underbrace{99...9}_{2019}$?

p2. For which positive integers n does there exist a triple of positive integers a, b, c, such that

$$LCM(a, b) + LCM(b, c) + LCM(a, c) = n?$$

Here $LCM(a, b)$ stands for the least common multiple of a, b.

p3. The polyhedron P is obtained by gluing together a tetrahedron with all edges equal to 1 and a pyramid with a square base and all edges also equal to 1. The gluing is done by aligning one of the faces of the tetrahedron with one of the triangular faces of the pyramid. How many faces does the polyhedron P have?
`https://cdn.artofproblemsolving.com/attachments/3/2/719d6632b1321e8a607722614e1de3ad7a889`
`png`

p4. An infection is spreading on an 8×8 chessboard. An infected cell remains infected forever, and on every step a healthy cell becomes infected if at least two of its neighbors are already infected, where two cells are called neighbors if they share a side. Can all cells become infected if initially
(1) 8 cells are infected?
(2) 7 cells are infected?

PS. You should use hide for answers.