Manhattan Mathematical Olympiad 2021

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– Grades 5-6

– **p1.** There is a piece of paper. On first step Sam cuts it into 7 pieces. On each next step Sam cuts one of the pieces into 4, 7 or 10 smaller pieces. Can Sam obtain 2021 pieces of paper after some number of steps?

p2. A $3 \times 3 \times 3$ cube is made out of 14 Red cubes and 13 Blue cubes. A row consists of 3 cubes aligned along one of the three directions. (There are 9 rows in each direction, that is, 27 rows in total).
(a) Can it happen that each row contains an odd number of Red cubes?
(b) Can it happen that each row contains an odd number of Blue cubes?

p3. Cut the shape below into two identical pieces. Pieces are called identical if one can put one on top of the other (flipping it over, perhaps) so that they match perfectly.
https://cdn.artofproblemsolving.com/attachments/d/3/0910a94af667d69266187b1372b6e606dab2a.png

p4. A class of 30 students is divided into 4 groups. Show that you can find two kids who are in the same group and have their 11^{th} birthdays on the same day of the week.

PS. You should use hide for answers.

– Grades 7-8

– **p1.** Show that $43^{101} + 23^{101}$ is divisible by 66.

p2. In Wonderland some towns are connected by railroads. The mirror country Rednowland has the same set of towns, but if two towns A and B in Wonderland are connected, then their mirror towns A' and B' in Rednowland are not connected, and if A and B are *not connected*, then A' and B' are connected. Alice of Wonderland has to make *at least* two stops to visit Queentown from her hometown. Show that Ecila of Rednowland can reach any town from any other town with at most two stops.

p3. In a right isosceles triangle ABC (with right angle B) the trisectors BM and BN and trisec-

tors AP and AQ are drawn. The trisector AQ (the one which is closer to the hypothenuse AC) intersects the trisectors BM and BN at points D and E (D is the one which is closer to A). Show that $AD = 2DE$.
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png

p4. Every 1×1 square in a 2021×2021 grid is colored with one of 4 colors in such a way that every 4 squares sharing a vertex are colored in different colors. Show that there is a row or a column which has only squares of 2 different colors in it.

PS. You should use hide for answers