

2021-22www.artofproblemsolving.com/community/c3001273

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- 1 (a) Find the value of the real number k , for which the polynomial $P(x) = x^3 - kx + 2$ has the number 2 as a root. In addition, for the value of k you will find, write this polynomial as the product of two polynomials with integer coefficients.
(b) If the positive real numbers a, b satisfy the equation

$$2a + b + \frac{4}{ab} = 10,$$

find the maximum possible value of a .

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- 2 Let ABC be an isosceles triangle, and point D in its interior such that

$$D\hat{B}C = 30^\circ, D\hat{B}A = 50^\circ, D\hat{C}B = 55^\circ$$

- (a) Prove that $\hat{B} = \hat{C} = 80^\circ$.
(b) Find the measure of the angle $D\hat{A}C$.

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- 3 On the board we write a series of n numbers, where $n \geq 40$, and each one of them is equal to either 1 or -1 , such that the following conditions both hold:

- (i) The sum of every 40 consecutive numbers is equal to 0.
(ii) The sum of every 42 consecutive numbers is not equal to 0.

We denote by S_n the sum of the n numbers of the board. Find the maximum possible value of S_n for all possible values of n .

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- 4 Find all couples of non-zero integers (x, y) such that, $x^2 + y^2$ is a common divisor of $x^5 + y$ and $y^5 + x$.
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