University of Washington - Math Hour Olympiad, an individual oral math Olympiad, https://sites.math.washington.edu
www.artofproblemsolving.com/community/c3002465
by parmenides51

2010.67 Round 1

p1. Is it possible to draw some number of diagonals in a convex hexagon so that every diagonal crosses EXACTLY three others in the interior of the hexagon? (Diagonals that touch at one of the corners of the hexagon DO NOT count as crossing.)

p2. A 3×3 square grid is filled with positive numbers so that
(a) the product of the numbers in every row is 1,
(b) the product of the numbers in every column is 1,
(c) the product of the numbers in any of the four 2×2 squares is 2.
What is the middle number in the grid? Find all possible answers and show that there are no others.

p3. Each letter in $HAGRID$'s name represents a distinct digit between 0 and 9. Show that

$$HAGRID \times H \times A \times G \times R \times I \times D$$

is divisible by 3. (For example, if $H = 1$, $A = 2$, $G = 3$, $R = 4$, $I = 5$, $D = 64$, then $HAGRID \times H \times A \times G \times R \times I \times D = 123456 \times 1 \times 2 \times 3 \times 4 \times 5 \times 6$).

p4. You walk into a room and find five boxes sitting on a table. Each box contains some number of coins, and you can see how many coins are in each box. In the corner of the room, there is a large pile of coins. You can take two coins at a time from the pile and place them in different boxes. If you can add coins to boxes in this way as many times as you like, can you guarantee that each box on the table will eventually contain the same number of coins?

p5. Alex, Bob and Chad are playing a table tennis tournament. During each game, two boys are playing each other and one is resting. In the next game the boy who lost a game goes to rest, and the boy who was resting plays the winner. By the end of tournament, Alex played a total of 10 games, Bob played 15 games, and Chad played 17 games. Who lost the second game?

Round 2

p6. After going for a swim in his vault of gold coins, Scrooge McDuck decides he wants to try to arrange some of his gold coins on a table so that every coin he places on the table touches exactly three others. Can he possibly do this? You need to justify your answer. (Assume the gold coins are circular, and that they all have the same size. Coins must be laid at on the table, and no two of them can overlap.)

p7. You have a deck of 50 cards, each of which is labeled with a number between 1 and 25. In the deck, there are exactly two cards with each label. The cards are shuffled and dealt to 25 students who are sitting at a round table, and each student receives two cards. The students will now play a game. On every move of the game, each student takes the card with the smaller number out of his or her hand and passes it to the person on his/her right. Each student makes this move at the same time so that everyone always has exactly two cards. The game continues until some student has a pair of cards with the same number. Show that this game will eventually end.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2011.67 Round 1

p1. In a chemical lab there are three vials: one that can hold 1 oz of fluid, another that can hold 2 oz, and a third that can hold 3 oz. The first is filled with grape juice, the second with sulfuric acid, and the third with water. There are also 3 empty vials in the cupboard, also of sizes 1 oz, 2 oz, and 3 oz. In order to save the world with grape-flavored acid, James Bond must make three full bottles, one of each size, filled with a mixture of all three liquids so that each bottle has the same ratio of juice to acid to water. How can he do this, considering he was silly enough not to bring any equipment?

p2. Twelve people, some are knights and some are knaves, are sitting around a table. Knaves always lie and knights always tell the truth. At some point they start up a conversation. The first person says, "There are no knights around this table." The second says, "There is at most one knight at this table." The third – "There are at most two knights at the table." And so on until the 12th says, "There are at most eleven knights at the table." How many knights are at the table? Justify your answer.

p3. Aquaman has a barrel divided up into six sections, and he has placed a red herring in each. Aquaman can command any fish of his choice to either 'jump counterclockwise to the next sector' or 'jump clockwise to the next sector.' Using a sequence of exactly 30 of these commands, can he relocate all the red herrings to one sector? If yes, show how. If no, explain why not.
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`png`

p4. Is it possible to place 13 integers around a circle so that the sum of any 3 adjacent numbers is exactly 13?

p5. Two girls are playing a game. The first player writes the letters A or B in a row, left to right, adding one letter on her turn. The second player switches any two letters after each move by the first player (the letters do not have to be adjacent), or does nothing, which also counts as a move. The game is over when each player has made 2011 moves. Can the second player plan her moves so that the resulting letters form a palindrome? (A palindrome is a sequence that reads the same forward and backwards, e.g. $AABABAA$.)

Round 2

p6. Eight students participated in a math competition. There were eight problems to solve. Each problem was solved by exactly five people. Show that there are two students who solved all eight problems between them.

p7. There are $3n$ checkers of three different colors: n red, n green and n blue. They were used to randomly fill a board with 3 rows and n columns so that each square of the board has one checker on it. Prove that it is possible to reshuffle the checkers within each row so that in each column there are checkers of all three colors. Moving checkers to a different row is not allowed.

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2012.57 Round 1

p1. Tom and Jerry stole a chain of 7 sausages and are now trying to divide the bounty. They take turns biting the sausages at one of the connections. When one of them breaks a connection, he may eat any single sausages that may fall out. Tom takes the first bite. Each of them is trying his best to eat more sausages than his opponent. Who will succeed?

p2. The King of the Mountain Dwarves wants to light his underground throne room by placing several torches so that the whole room is lit. The king, being very miserly, wants to use as few torches as possible. What is the least number of torches he could use? (You should show why he can't do it with a smaller number of torches.)
This is the shape of the throne room:
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png
Also, the walls in all rooms are lined with velvet and do not reflect the light. For example, the

picture on the right shows how another room in the castle is partially lit.
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png

p3. In the Hundred Acre Wood, all the animals are either knights or liars. Knights always tell the truth and liars always lie. One day in the Wood, Winnie-the-Pooh, a knight, decides to visit his friend Rabbit, also a noble knight. Upon arrival, Pooh finds his friend sitting at a round table with 5 other guests.
One-by-one, Pooh asks each person at the table how many of his two neighbors are knights. Surprisingly, he gets the same answer from everybody! ”Oh bother!” proclaims Pooh. ”I still don't have enough information to figure out how many knights are at this table.”
”But it's my birthday,” adds one of the guests. ”Yes, it's his birthday!” agrees his neighbor.
Now Pooh can tell how many knights are at the table. Can you?

p4. Several girls participate in a tennis tournament in which each player plays each other player exactly once. At the end of the tournament, it turns out that each player has lost at least one of her games. Prove that it is possible to find three players A, B, and C such that A defeated B, B defeated C, and C defeated A.

p5. There are 40 piles of stones with an equal number of stones in each. Two players, Ann and Bob, can select any two piles of stones and combine them into one bigger pile, as long as this pile would not contain more than half of all the stones on the table. A player who can't make a move loses. Ann goes first. Who wins?

Round 2

p6. In a galaxy far, far away, there is a United Galactic Senate with 100 Senators. Each Senator has no more than three enemies. Tired of their arguments, the Senators want to split into two parties so that each Senator has no more than one enemy in his own party. Prove that they can do this. (Note: If A is an enemy of B, then B is an enemy of A.)

p7. Harry has a 2012 by 2012 chessboard and checkers numbered from 1 to 2012×2012. Can he place all the checkers on the chessboard in such a way that whatever row and column Professor Snape picks, Harry will be able to choose three checkers from this row and this column such that the product of the numbers on two of the checkers will be equal to the number on the third?
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PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2013.67 Round 1

p1. Goldilocks enters the home of the three bears – Papa Bear, Mama Bear, and Baby Bear. Each bear is wearing a different-colored shirt – red, green, or blue. All the bears look the same to Goldilocks, so she cannot otherwise tell them apart.
The bears in the red and blue shirts each make one true statement and one false statement.
The bear in the red shirt says: "I'm Blue's dad. I'm Green's daughter."
The bear in the blue shirt says: "Red and Green are of opposite gender. Red and Green are my parents."
Help Goldilocks find out which bear is wearing which shirt.

p2. The University of Washington is holding a talent competition. The competition has five contests: math, physics, chemistry, biology, and ballroom dancing. Any student can enter into any number of the contests but only once for each one. For example, a student may participate in math, biology, and ballroom.
It turned out that each student participated in an odd number of contests. Also, each contest had an odd number of participants. Was the total number of contestants odd or even?

p3. The 99 greatest scientists of Mars and Venus are seated evenly around a circular table. If any scientist sees two colleagues from her own planet sitting an equal number of seats to her left and right, she waves to them. For example, if you are from Mars and the scientists sitting two seats to your left and right are also from Mars, you will wave to them. Prove that at least one of the 99 scientists will be waving, no matter how they are seated around the table.

p4. One hundred boys participated in a tennis tournament in which every player played each other player exactly once and there were no ties. Prove that after the tournament, it is possible for the boys to line up for pizza so that each boy defeated the boy standing right behind him in line.

p5. To celebrate space exploration, the Science Fiction Museum is going to read Star Wars and Star Trek stories for 24 hours straight. A different story will be read each hour for a total of 12 Star Wars stories and 12 Star Trek stories. George and Gene want to listen to exactly 6 Star Wars and 6 Star Trek stories. Show that no matter how the readings are scheduled, the friends can find a block of 12 consecutive hours to listen to the stories together.

Round 2

p6. 2013 people attended Cinderella's ball. Some of the guests were friends with each other. At midnight, the guests started turning into mice. After the first minute, everyone who had no friends at the ball turned into a mouse. After the second minute, everyone who had exactly one friend among the remaining people turned into a mouse. After the third minute, everyone who had two human friends left in the room turned into a mouse, and so on. What is the maximal number of people that could have been left at the ball after 2013 minutes?

p7. Bill and Charlie are playing a game on an infinite strip of graph paper. On Bill's turn, he marks two empty squares of his choice (not necessarily adjacent) with crosses. Charlie, on his turn, can erase any number of crosses, as long as they are all adjacent to each other. Bill wants to create a line of 2013 crosses in a row. Can Charlie stop him?

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2014.57 Round 1

p1. Three snails – Alice, Bobby, and Cindy – were racing down a road.
Whenever one snail passed another, it waved at the snail it passed.
During the race, Alice waved 3 times and was waved at twice.
Bobby waved 4 times and was waved at 3 times.
Cindy waved 5 times. How many times was she waved at?

p2. Sherlock and Mycroft are playing Battleship on a 4×4 grid. Mycroft hides a single 3×1 cruiser somewhere on the board. Sherlock can pick squares on the grid and fire upon them. What is the smallest number of shots Sherlock has to fire to guarantee at least one hit on the cruiser?

p3. Thirty girls – 13 of them in red dresses and 17 in blue dresses – were dancing in a circle, hand-in-hand. Afterwards, each girl was asked if the girl to her right was in a blue dress. Only the girls who had both neighbors in red dresses or both in blue dresses told the truth. How many girls could have answered "Yes"?

p4. Herman and Alex play a game on a 5×5 board. On his turn, a player can claim any open square as his territory. Once all the squares are claimed, the winner is the player whose territory has the longer border. Herman goes first. If both play their best, who will win, or will the game end in a draw?
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png

p5. Is it possible to find 2014 distinct positive integers whose sum is divisible by each of them?

Round 2

p6. Hermione and Ron play a game that starts with 129 hats arranged in a circle. They take turns magically transforming the hats into animals. On each turn, a player picks a hat and chooses whether to change it into a badger or into a raven. A player loses if after his or her turn there are two animals of the same species right next to each other. Hermione goes first. Who loses?

p7. Three warring states control the corner provinces of the island whose map is shown below.
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png
As a result of war, each of the remaining 18 provinces was occupied by one of the states. None of the states was able to occupy any province on the coast opposite their corner. The states would like to sign a peace treaty. To do this, they each must send ambassadors to a place where three provinces, one controlled by each state, come together. Prove that they can always find such a place to meet.
For example, if the provinces are occupied as shown here, the squares mark possible meeting spots.
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PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2015.57 Round 1

p1. A party is attended by ten people (men and women). Among them is Pat, who always lies to people of the opposite gender and tells the truth to people of the same gender.
Pat tells five of the guests: "There are more men than women at the party."
Pat tells four of the guests: "There are more women than men at the party."
Is Pat a man or a woman?

p2. Once upon a time in a land far, far away there lived 100 knights, 99 princesses, and 101 dragons. Over time, knights beheaded dragons, dragons ate princesses, and princesses poisoned knights. But they always obeyed an ancient law that prohibits killing any creature who has killed an odd number of others. Eventually only one creature remained alive. Could it have been a knight? A dragon? A princess?

p3. The numbers $1 \circ 2 \circ 3 \circ 4 \circ 5 \circ 6 \circ 7 \circ 8 \circ 9 \circ 10$ are written in a line. Alex and Vicky play a game, taking turns inserting either an addition or a multiplication symbol between adjacent numbers. The last player to place a symbol wins if the resulting expression is odd and loses if it is even. Alex moves first. Who wins?
(Remember that multiplication is performed before addition.)

p4. A chess tournament had 8 participants. Each participant played each other participant once. The winner of a game got 1 point, the loser 0 points, and in the case of a draw each got $1/2$ a point. Each participant scored a different number of points, and the person who got 2nd prize scored the same number of points as the 5th, 6th, 7th and 8th place participants combined. Can you determine the result of the game between the 3rd place player and the 5th place player?

p5. One hundred gnomes sit in a circle. Each gnome gets a card with a number written on one side and a different number written on the other side. Prove that it is possible for all the gnomes to lay down their cards so that no two neighbors have the same numbers facing up.

Round 2

p6. A casino machine accepts tokens of 32 different colors, one at a time. For each color, the player can choose between two fixed rewards. Each reward is up to $\$10$ cash, plus maybe another token. For example, a blue token always gives the player a choice of getting either $\$5$ plus a red token or $\$3$ plus a yellow token; a black token can always be exchanged either for $\$10$ (but no token) or for a brown token (but no cash). A player may keep playing as long as he has a token. Rob and Bob each have one white token. Rob watches Bob play and win $\$500$. Prove that Rob can win at least $\$1000$.
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`png`

p7. Each of the 100 residents of Pleasantville has at least 30 friends in town. A resident votes in the mayoral election only if one of her friends is a candidate. Prove that it is possible to nominate two candidates for mayor so that at least half of the residents will vote.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2016.67 Round 1

p1. At a fortune-telling exam, 13 witches are sitting in a circle. To pass the exam, a witch must correctly predict, for everybody except herself and her two neighbors, whether they will pass or fail. Each witch predicts that each of the 10 witches she is asked about will fail. How many witches could pass?

p2. Out of 152 coins, 7 are counterfeit. All counterfeit coins have the same weight, and all real coins have the same weight, but counterfeit coins are lighter than real coins. How can you find 19 real coins if you are allowed to use a balance scale three times?

p3. The digits of a number N increase from left to right. What could the sum of the digits of $9 \times N$ be?

p4. The sides and diagonals of a pentagon are colored either blue or red. You can choose three vertices and flip the colors of all three lines that join them. Can every possible coloring be turned all blue by a sequence of such moves?
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p5. You have 100 pancakes, one with a single blueberry, one with two blueberries, one with three blueberries, and so on. The pancakes are stacked in a random order. Count the number of blueberries in the top pancake and call that number N. Pick up the stack of the top N pancakes and flip it upside down. Prove that if you repeat this counting-and-flipping process, the pancake with one blueberry will eventually end up at the top of the stack.

Round 2

p6. A circus owner will arrange 100 fleas on a long string of beads, each flea on her own bead. Once arranged, the fleas start jumping using the following rules. Every second, each flea chooses the closest bead occupied by one or more of the other fleas, and then all fleas jump simultaneously to their chosen beads. If there are two places where a flea could jump, she jumps to the right. At the start, the circus owner arranged the fleas so that, after some time, they all gather on just two beads. What is the shortest amount of time it could take for this to happen?

p7. The faraway land of Noetheria has 2016 cities. There is a nonstop flight between every pair of cities. The price of a nonstop ticket is the same in both directions, but flights between different pairs of cities have different prices. Prove that you can plan a route of 2015 consecutive flights so that each flight is cheaper than the previous one. It is permissible to visit the same city several times along the way.

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/

community/c5h2760506p24143309).

2017.67 Round 1

p1. Ten children arrive at a birthday party and leave their shoes by the door. All the children have different shoe sizes. Later, as they leave one at a time, each child randomly grabs a pair of shoes their size or larger. After some kids have left, all of the remaining shoes are too small for any of the remaining children. What is the greatest number of shoes that might remain by the door?

p2. Turans, the king of Saturn, invented a new language for his people. The alphabet has only 6 letters: A, N, R, S, T, U; however, the alphabetic order is different than in English. A word is any sequence of 6 different letters. In the dictionary for this language, the first word is SATURN. Which word follows immediately after TURANS?

p3. Benji chooses five integers. For each pair of these numbers, he writes down the pair's sum. Can all ten sums end with different digits?

p4. Nine dwarves live in a house with nine rooms arranged in a 3×3 square. On Monday morning, each dwarf rubs noses with the dwarves in the adjacent rooms that share a wall. On Monday night, all the dwarves switch rooms. On Tuesday morning, they again rub noses with their adjacent neighbors. On Tuesday night, they move again. On Wednesday morning, they rub noses for the last time. Show that there are still two dwarves who haven't rubbed noses with one another.

p5. Anna and Bobby take turns placing rooks in any empty square of a pyramid-shaped board with 100 rows and 200 columns. If a player places a rook in a square that can be attacked by a previously placed rook, he or she loses. Anna goes first. Can Bobby win no matter how well Anna plays?
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Round 2

p6. Some boys and girls, all of different ages, had a snowball fight. Each girl threw one snowball at every kid who was older than her. Each boy threw one snowball at every kid who was younger than him. Three friends were hit by the same number of snowballs, and everyone else took fewer hits than they did. Prove that at least one of the three is a girl.

p7. Last year, jugglers from around the world travelled to Jakarta to participate in the Jubilant

Juggling Jamboree. The festival lasted 32 days, with six solo performances scheduled each day. The organizers noticed that for any two days, there was exactly one juggler scheduled to perform on both days. No juggler performed more than once on a single day. Prove there was a juggler who performed every day.

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2018.67 Round 1

p1. Alice and Bob played 25 games of rock-paper-scissors. Alice played rock 12 times, scissors 6 times, and paper 7 times. Bob played rock 13 times, scissors 9 times, and paper 3 times. If there were no ties, who won the most games?
(Remember, in each game each player picks one of rock, paper, or scissors. Rock beats scissors, scissors beat paper, and paper beats rock. If they choose the same object, the result is a tie.)

p2. On the planet Vulcan there are eight big volcanoes and six small volcanoes. Big volcanoes erupt every three years and small volcanoes erupt every two years. In the past five years, there were 30 eruptions. How many volcanoes could erupt this year?

p3. A tangle is a sequence of digits constructed by picking a number $N \ge 0$ and writing the integers from 0 to N in some order, with no spaces. For example, 010123459876 is a tangle with $N = 10$. A palindromic sequence reads the same forward or backward, such as 878 or 6226. The shortest palindromic tangle is 0. How long is the second-shortest palindromic tangle?

p4. Balls numbered 1 to N have been randomly arranged in a long input tube that feeds into the upper left square of an 8×8 board. An empty exit tube leads out of the lower right square of the board. Your goal is to arrange the balls in order from 1 to N in the exit tube. As a move, you may
1. move the next ball in line from the input tube into the upper left square of the board,
2. move a ball already on the board to an adjacent square to its right or below, or
3. move a ball from the lower right square into the exit tube.
No square may ever hold more than one ball. What is the largest number N for which you can achieve your goal, no matter how the balls are initially arranged? You can see the order of the balls in the input tube before you start.
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p5. A 2018×2018 board is covered by non-overlapping 2×1 dominoes, with each domino covering two squares of the board. From a given square, a robot takes one step to the other square of the domino it is on and then takes one more step in the same direction. Could the robot continue

moving this way forever without falling off the board?
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Round 2

p6. Seventeen teams participated in a soccer tournament where a win is worth 1 point, a tie is worth 0 points, and a loss is worth -1 point. Each team played each other team exactly once. At least $\frac{3}{4}$ of all games ended in a tie. Show that there must be two teams with the same number of points at the end of the tournament.

p7. The city of Old Haven is known for having a large number of secret societies. Any person may be a member of multiple societies. A secret society is called influential if its membership includes at least half the population of Old Haven. Today, there are 2018 influential secret societies. Show that it is possible to form a council of at most 11 people such that each influential secret society has at least one member on the council.

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2019.67 Round 1

p1. Three two-digit numbers are written on a board. One starts with 5, another with 6, and the last one with 7. Annie added the first and the second numbers; Benny added the second and the third numbers; Denny added the third and the first numbers. Could it be that one of these sums is equal to 148, and the two other sums are three-digit numbers that both start with 12?

p2. Three rocks, three seashells, and one pearl are placed in identical boxes on a circular plate in the order shown. The lids of the boxes are then closed, and the plate is secretly rotated. You can open one box at a time. What is the smallest number of boxes you need to open to know where the pearl is, no matter how the plate was rotated?
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p3. Two detectives, Holmes and Watson, are hunting the thief Raffles in a library, which has the floorplan exactly as shown in the diagram. Holmes and Watson start from the center room marked D. Show that no matter where Raffles is or how he moves, Holmes and Watson can find him. Holmes and Watson do not need to stay together. A detective sees Raffles only if they are in the same room. A detective cannot stand in a doorway to see two rooms at the same time.

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png

p4. A museum has a 4×4 grid of rooms. Every two rooms that share a wall are connected by a door. Each room contains some paintings. The total number of paintings along any path of 7 rooms from the lower left to the upper right room is always the same. Furthermore, the total number of paintings along any path of 7 rooms from the lower right to the upper left room is always the same. The guide states that the museum has exactly 500 paintings. Show that the guide is mistaken.
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png

p5. The numbers 1–14 are placed around a circle in some order. You can swap two neighbors if they differ by more than 1. Is it always possible to rearrange the numbers using swaps so they are ordered clockwise from 1 to 14?

Round 2

p6. A triangulation of a regular polygon is a way of drawing line segments between its vertices so that no two segments cross, and the interior of the polygon is divided into triangles. A flip move erases a line segment between two triangles, creating a quadrilateral, and replaces it with the opposite diagonal through that quadrilateral. This results in a new triangulation.
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png
Given any two triangulations of a polygon, is it always possible to find a sequence of flip moves that transforms the first one into the second one?
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png

p7. Is it possible to place the numbers from 1 to 121 in an 11×11 table so that numbers that differ by 1 are in horizontally or vertically adjacent cells and all the perfect squares $(1, 4, 9, ..., 121)$ are in one column?

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2022.67 Round 1

p1. Nineteen witches, all of different heights, stand in a circle around a campfire. Each witch says whether she is taller than both of her neighbors, shorter than both, or in-between. Exactly three said "I am taller." How many said "I am in-between"?

p2. Alex is writing a sequence of A's and B's on a chalkboard. Any 20 consecutive letters must have an equal number of A's and B's, but any 22 consecutive letters must have a different number of A's and B's. What is the length of the longest sequence Alex can write?.

p3. A police officer patrols a town whose map is shown. The officer must walk down every street segment at least once and return to the starting point, only changing direction at intersections and corners. It takes the officer one minute to walk each segment. What is the fastest the officer can complete a patrol?
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p4. A zebra is a new chess piece that jumps in the shape of an "L" to a location three squares away in one direction and two squares away in a perpendicular direction. The picture shows all the moves a zebra can make from its given position. Is it possible for a zebra to make a sequence of 64 moves on an 8×8 chessboard so that it visits each square exactly once and returns to its starting position?
https://cdn.artofproblemsolving.com/attachments/2/d/01a8af0214a2400b279816fc5f6c039320e81
png

p5. Ann places the integers $1, 2, ..., 100$ in a 10×10 grid, however she wants. In each round, Bob picks a row or column, and Ann sorts it from lowest to highest (left-to-right for rows; top-to-bottom for columns). However, Bob never sees the grid and gets no information from Ann.
After eleven rounds, Bob must name a single cell that is guaranteed to contain a number that is at least 30 and no more than 71. Can he find a strategy to do this, no matter how Ann originally arranged the numbers?

<u>Round 2</u>

p6. Evelyn and Odette are playing a game with a deck of 101 cards numbered 1 through 101. At the start of the game the deck is split, with Evelyn taking all the even cards and Odette taking all the odd cards. Each shuffles her cards. On every move, each player takes the top card from her deck and places it on a table. The player whose number is higher takes both cards from the table and adds them to the bottom of her deck, first the opponent's card, then her own. The first player to run out of cards loses.
Card 101 was played against card 2 on the 10th move. Prove that this game will never end.
https://cdn.artofproblemsolving.com/attachments/8/1/aa16fe1fb4a30d5b9e89ac53bdae0d1bdf20b
png

p7. The Vogon spaceship Tempest is descending on planet Earth. It will land on five adjacent buildings within a 10×10 grid, crushing any teacups on roofs of buildings within a 5×1 length of blocks (vertically or horizontally). As Commander of the Space Force, you can place any number of teacups on rooftops in advance. When the ship lands, you will hear how many teacups the spaceship breaks, but not where they were. (In the figure, you would hear 4 cups break.)
What is the smallest number of teacups you need to place to ensure you can identify at least one building the spaceship landed on?
https://cdn.artofproblemsolving.com/attachments/8/7/2a48592b371bba282303e60b4ff38f42de355
png

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2023.67 Round 1

p1. Ash is running around town catching Pokémon. Each day, he may add $3, 4$, or 5 Pokémon to his collection, but he can never add the same number of Pokémon on two consecutive days. What is the smallest number of days it could take for him to collect exactly 100 Pokémon?

p2. Jack and Jill have ten buckets. One bucket can hold up to 1 gallon of water, another can hold up to 2 gallons, and so on, with the largest able to hold up to 10 gallons. The ten buckets are arranged in a line as shown below. Jack and Jill can pour some amount of water into each bucket, but no bucket can have less water than the one to its left. Is it possible that together, the ten buckets can hold 36 gallons of water?
https://cdn.artofproblemsolving.com/attachments/f/8/0b6524bebe8fe859fe7b1bc887ac786106fc1
png

p3. There are 2023 knights and liars standing in a row. Knights always tell the truth and liars always lie. Each of them says, "the number of liars to the left of me is greater than the number of knights to the right." How many liars are there?

p4. Camila has a deck of 101 cards numbered $1, 2, ..., 101$. She starts with 50 random cards in her hand and the rest on a table with the numbers visible. In an exchange, she replaces all 50 cards in her hand with her choice of 50 of the 51 cards from the table. Show that Camila can make at most 50 exchanges and end up with cards $1, 2, ..., 50.$
https://cdn.artofproblemsolving.com/attachments/0/6/c89e65118764f3b593da45264bfd0d89e9506
png

p5. There are 101 pirates on a pirate ship: the captain and 100 crew. Each pirate, including the captain, starts with 1 gold coin. The captain makes proposals for redistributing the coins, and the crew vote on these proposals. The captain does not vote. For every proposal, each crew member greedily votes "yes" if he gains coins as a result of the proposal, "no" if he loses coins, and passes otherwise. If strictly more crew members vote ”yes” than ”no,” the proposal takes effect. The captain can make any number of proposals, one after the other. What is the largest number of coins the captain can accumulate?

Round 2

p6. The town of Lumenville has 100 houses and is preparing for the math festival. The Tesla wiring company will lay lengths of power wire in straight lines between the houses so that power flows between any two houses, possibly by passing through other houses. The Edison lighting company will hang strings of lights in straight lines between pairs of houses so that each house is connected by a string to exactly one other. Show that however the houses are arranged, the Edison company can always hang their strings of lights so that the total length of the strings is no more than the total length of the power wires the Tesla company used.
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png

p7. You are given a sequence of 16 digits. Is it always possible to select one or more digits in a row, so that multiplying them results in a square number?
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png

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).