

EGMO Team Selection Test for Brazil 2022

www.artofproblemsolving.com/community/c3003199

by parmenides51, outback, Hoapham235, mathisreal

- 1 Let a, b, c be positive real numbers such that:

$$ab - c = 3$$

$$abc = 18$$

Calculate the numerical value of $\frac{ab}{c}$

- 2 Let $\triangle ABC$ be a triangle in which $\angle ACB = 40^\circ$ and $\angle BAC = 60^\circ$. Let D be a point inside the segment BC such that $CD = \frac{AB}{2}$ and let M be the midpoint of the segment AC . How much is the angle $\angle CMD$ in degrees?

- 3 A natural number is called *chaotigal* if it and its successor both have the sum of their digits divisible by 2021. How many digits are in the smallest chaotigal number?

- 4 Mariana plays with an 8×8 board with all its squares blank. She says that two houses are *neighbors* if they have a common side or vertex, that is, two houses can be neighbors vertically, horizontally or diagonally. The game consists of filling the 64 squares on the board, one after the other, each with a number according to the following rule: she always chooses a house blank and fill it with an integer equal to the number of neighboring houses that are still in White. Once this is done, the house is no longer considered blank.

Show that the value of the sum of all 64 numbers written on the board at the end of the game does not depend in the order of filling. Also, calculate the value of this sum.

Note: A house is not neighbor to itself.

Mariana brinca com um tabuleiro 8×8 com todas as suas casas em branco. Ela diz que duas casas s~ao vizinhas se elas possu~ırem um lado ou um v~ertice em comum, ou seja, duas casas podem ser vizinhas

verticalmente, horizontalmente ou diagonalmente. A brincadeira consiste em preencher as 64 casas do tabuleiro,

uma ap~os a outra, cada uma com um n~umero de acordo com a seguinte regra: ela escolhe sempre uma casa

em branco e a preenche com o n~umero inteiro igual a quantidade de casas vizinhas desta que ainda estejam em

branco. Feito isso, a casa n~ao ~e mais considerada em branco.

Demonstre que o valor da soma de todos os 64 n~umeros escritos no tabuleiro ao final da brincadeira n~ao depende

da ordem do preenchimento. Al´em disso, calcule o valor dessa soma.
Observa,c˜ao: Uma casa n˜ao ´e vizinha a si mesma

-
- 5** For a given value t , we consider number sequences a_1, a_2, a_3, \dots such that $a_{n+1} = \frac{a_n+t}{a_n+1}$ for all $n \geq 1$.
- (a) Suppose that $t = 2$. Determine all starting values $a_1 > 0$ such that $\frac{4}{3} \leq a_n \leq \frac{3}{2}$ holds for all $n \geq 2$.
- (b) Suppose that $t = -3$. Investigate whether $a_{2020} = a_1$ for all starting values a_1 different from -1 and 1 .
-
- 6** The diagonals AC and BD of a convex quadrilateral $ABCD$ intersect at point M . The bisector of $\angle ACD$ meets the ray BA at K . Given that $MA \cdot MC + MA \cdot CD = MB \cdot MD$, prove that $\angle BKC = \angle CDB$.
-
- 7** Let a_1, a_2, \dots, a_{2n} be $2n$ elements of $\{1, 2, 3, \dots, 2n-1\}$ ($n > 3$) with the sum $a_1 + a_2 + \dots + a_{2n} = 4n$. Prove that exist some numbers a_i with the sum is $2n$.
-
- 8** Find all pairs (a, b) of positive integers, such that for **every** n positive integer, the equality $a^n + b^n = c_n^{n+1}$ is true, for some c_n positive integer.
-