Mid-Michigan Mathematical Olympiad, Grades 5-6
www.artofproblemsolving.com/community/c3003999
by parmenides51

2002 **p1.** Find all triples of positive integers such that the sum of their reciprocals is equal to one.

p2. Prove that $a(a+1)(a+2)(a+3)$ is divisible by 24.

p3. There are 20 very small red chips and some blue ones. Find out whether it is possible to put them on a large circle such that
(a) for each chip positioned on the circle the antipodal position is occupied by a chip of different color;
(b) there are no two neighboring blue chips.

p4. A 12 liter container is filled with gasoline. How to split it in two equal parts using two empty 5 and 8 liter containers?

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2003 **p1.** One day, Granny Smith bought a certain number of apples at Horock's Farm Market. When she returned the next day she found that the price of the apples was reduced by 20%. She could therefore buy more apples while spending the same amount as the previous day. How many percent more?

p2. You are asked to move several boxes. You know nothing about the boxes except that each box weighs no more than 10 tons and their total weight is 100 tons. You can rent several trucks, each of which can carry no more than 30 tons. What is the minimal number of trucks you can rent and be sure you will be able to carry all the boxes at once?

p3. The five numbers $1, 2, 3, 4, 5$ are written on a piece of paper. You can select two numbers and increase them by 1. Then you can again select two numbers and increase those by 1. You can repeat this operation as many times as you wish. Is it possible to make all numbers equal?

p4. There are 15 people in the room. Some of them are friends with others. Prove that there is a person who has an even number of friends in the room.

Bonus Problem

p5. Several ants are crawling along a circle with equal constant velocities (not necessarily in the same direction). If two ants collide, both immediately reverse direction and crawl with the same velocity. Prove that, no matter how many ants and what their initial positions are, they will, at some time, all simultaneously return to the initial positions.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2004 **p1.** On the island of Nevermind some people are liars; they always lie. The remaining habitants of the island are truthlovers; they tell only the truth. Three habitants of the island, A, B, and C met this morning. A said: "All of us are liars". B said: "Only one of us is a truthlover".
Who of them is a liar and who of them is a truthlover?

p2. Pinocchio has 9 pieces of paper. He is allowed to take a piece of paper and cut it in 5 pieces or 7 pieces which increases the number of his pieces. Then he can take again one of his pieces of paper and cut it in 5 pieces or 7 pieces. He can do this again and again as many times as he wishes. Can he get 2004 pieces of paper?

p3. In Dragonland there are coins of 1 cent, 2 cents, 10 cents, 20 cents, and 50 cents. What is the largest amount of money one can have in coins, yet still not be able to make exactly 1 dollar?

p4. Find all solutions a, b, c, d, e if it is known that they represent distinct digits and satisfy the following:

$$\begin{array}{ccccc} & a & b & c & d \\ + & a & c & a & c \\ \hline c & d & e & b & c \end{array}$$

p5. Two players play the following game. On the lowest left square of an 8×8 chessboard there is a rook. The first player is allowed to move the rook up or to the right by an arbitrary number of squares. The second player is also allowed to move the rook up or to the right by an arbitrary number of squares. Then the first player is allowed to do this again, and so on. The one who moves the rook to the upper right square wins. Who has a winning strategy?

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2005 **p1.** Is there an integer such that the product of all whose digits equals 99 ?

p2. An elevator in a 100 store building has only two buttons: UP and DOWN. The UP button makes the elevator go 13 floors up, and the DOWN button makes it go 8 floors down. Is it possible to go from the 13th floor to the 8th floor?

p3. Cut the triangle shown in the picture into three pieces and rearrange them into a rectangle. (Pieces can not overlap.)
`https://cdn.artofproblemsolving.com/attachments/9/f/359d3b987012de1f3318c3f06710daabe66f2`
`png`

p4. Two players Tom and Sid play the following game. There are two piles of rocks, 5 rocks in the first pile and 6 rocks in the second pile. Each of the players in his turn can take either any amount of rocks from one pile or the same amount of rocks from both piles. The winner is the player who takes the last rock. Who does win in this game if Tom starts the game?

p5. In the next long multiplication example each letter encodes its own digit. Find these digits.

			a	b
*			c	d
		c	e	f
+		a	b	
	c	f	d	f

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2006 **p1.** Find all solutions a, b, c, d, e, f if it is known that they represent distinct digits and satisfy the following:

	a	b	c	a
+		d	d	e
			d	e
d	f	f	d	d

p2. Snowhite wrote on a piece of paper a whole number greater than 1 and multiplied it by itself. She obtained a number, all digits of which are 1: $n^2 = 111...111$ Does she know how to multiply?

p3. Two players play the following game on an 8×8 chessboard. The first player can put a bishop on an arbitrary square. Then the second player can put another bishop on a free square that is not controlled by the first bishop. Then the first player can put a new bishop on a free square that is not controlled by the bishops on the board. Then the second player can do the same, etc. A player who cannot put a new bishop on the board loses the game. Who has a winning strategy?

p4. Four girls Marry, Jill, Ann and Susan participated in the concert. They sang songs. Every song was performed by three girls. Mary sang 8 songs, more then anybody. Susan sang 5 songs less then all other girls. How many songs were performed at the concert?

p5. Pinocchio has a 10×10 table of numbers. He took the sums of the numbers in each row and each such sum was positive. Then he took the sum of the numbers in each columns and each such sum was negative. Can you trust Pinocchio's calculations?

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2007 **p1.** The Evergreen School booked buses for a field trip. Altogether, 138 people went to West Lake, while 115 people went to East Lake. The buses all had the same number of seats, and every bus has more than one seat. All seats were occupied and everybody had a seat. How many seats were there in each bus?

p2. In New Scotland there are three kinds of coins: 1 cent, 6 cent, and 36 cent coins. Josh has 50 of the 36-cent coins (and no other coins). He is allowed to exchange a 36 cent coin for 6 coins of 6 cents, and to exchange a 6 cent coin for 6 coins of 1 cent. Is it possible that after several exchanges Josh will have 150 coins?

p3. Pinocchio multiplied two 2 digit numbers. But witch Masha erased some of the digits. The erased digits are the ones marked with a $*$. Could you help Pinocchio to restore all the erased digits?

$$\begin{array}{ccccc}
 & & & 9 & 5 \\
\text{x} & & & * & * \\
\hline
 & & & * & * \\
+ & 1 & * & * & \\
\hline
 & * & * & * & *
\end{array}$$

Find all solutions.

p4. There are 50 senators and 435 members of House of Representatives. On Friday all of them voted a very important issue. Each senator and each representative was required to vote either "yes" or "no". The announced results showed that the number of "yes" votes was greater than the number of "no" votes by 24. Prove that there was an error in counting the votes.

p5. Was there a year in the last millennium (from 1000 to 2000) such that the sum of the digits of that year is equal to the product of the digits?

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/

community/c5h2760506p24143309).

2008 **p1.** Insert "$+$" signs between some of the digits in the following sequence to obtain correct equality:

$$1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7 = 100$$

p2. A square is tiled by smaller squares as shown in the figure. Find the area of the black square in the middle if the perimeter of the big square $ABCD$ is 40 cm.
https://cdn.artofproblemsolving.com/attachments/8/c/d54925cba07f63ec8578048f46e1e730cb8df
png

p3. Jack made 3 quarts of fruit drink from orange and apple juice. $\frac{2}{5}$ of his drink is orange juice and the rest is apple juice. Nick prefers more orange juice in the drink. How much orange juice should he add to the drink to obtain a drink composed of $\frac{3}{5}$ of orange juice?

p4. A train moving at 55 miles per hour meets and is passed by a train moving moving in the opposite direction at 35 miles per hour. A passenger in the first train sees that the second train takes 8 seconds to pass him. How long is the second train?

p5. It is easy to arrange 16 checkers in 10 rows of 4 checkers each, but harder to arrange 9 checkers in 10 rows of 3 checkers each. Do both.

p6. Every human that lived on Earth exchanged some number of handshakes with other humans. Show that the number of people that made an odd number of handshakes is even.

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2009 **p1.** Anne purchased yesterday at WalMart in Puerto Rico 6 identical notebooks, 8 identical pens and 7 identical erasers. Anne remembers that each eraser costs 73 cents. She did not buy anything else. Anne told her mother that she spent 12 dollars and 76 cents at Walmart. Can she be right? Note that in Puerto Rico there is no sales tax.

p2. Two men ski one after the other first in a flat field and then uphill. In the field the men run with the same velocity 12 kilometers/hour. Uphill their velocity drops to 8 kilometers/hour. When both skiers enter the uphill trail segment the distance between them is 300 meters less than the initial distance in the field. What was the initial distance between skiers? (There are 1000 meters in 1 kilometer.)

p3. In the equality $** + **** = ****$ all the digits are replaced by $*$. Restore the equality if it is known that any numbers in the equality does not change if we write all its digits in the opposite order.

p4. If a polyleg has even number of legs he always tells truth. If he has an odd number of legs he always lies. Once a green polyleg told a dark-blue polyleg "- I have 8 legs. And you have only 6 legs!" The offended dark-blue polyleg replied "-It is me who has 8 legs, and you have only 7 legs!" A violet polyleg added "-The dark-blue polyleg indeed has 8 legs. But I have 9 legs!" Then a stripped polyleg started: "-None of you has 8 legs. Only I have 8 legs!" Which polyleg has exactly 8 legs?

p5. Cut the figure shown below in two equal pieces. (Both the area and the form of the pieces must be the same.) https://cdn.artofproblemsolving.com/attachments/e/4/778678c1e8748e213ffc9 png

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2010 **p1.** Ben and his dog are walking on a path around a lake. The path is a loop 500 meters around. Suddenly the dog runs away with velocity 10 km/hour. Ben runs after it with velocity 8 km/hour. At the moment when the dog is 250 meters ahead of him, Ben turns around and runs at the same speed in the opposite direction until he meets the dog. For how many minutes does Ben run?

p2. The six interior angles in two triangles are measured. One triangle is obtuse (i.e. has an angle larger than 90^o) and the other is acute (all angles less than 90^o). Four angles measure 120^o, 80^o, 55^o and 10^o. What is the measure of the smallest angle of the acute triangle?

p3. The figure below shows a 10×10 square with small 2×2 squares removed from the corners. What is the area of the shaded region?
https://cdn.artofproblemsolving.com/attachments/7/5/a829487cc5d937060e8965f6da3f4744ba558 png

p4. Two three-digit whole numbers are called relatives if they are not the same, but are written using the same triple of digits. For instance, 244 and 424 are relatives. What is the minimal number of relatives that a three-digit whole number can have if the sum of its digits is 10?

p5. Three girls, Ann, Kelly, and Kathy came to a birthday party. One of the girls wore a red dress, another wore a blue dress, and the last wore a white dress. When asked the next day, one girl said that Kelly wore a red dress, another said that Ann did not wear a red dress, the last said that Kathy did not wear a blue dress. One of the girls was truthful, while the other two lied. Which statement was true?

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2012 **p1.** A boy has as many sisters as brothers. How ever, his sister has twice as many brothers as sisters. How many boys and girls are there in the family?

p2. Solve each of the following problems.
(1) Find a pair of numbers with a sum of 11 and a product of 24.
(2) Find a pair of numbers with a sum of 40 and a product of 400.
(3) Find three consecutive numbers with a sum of 333.
(4) Find two consecutive numbers with a product of 182.

p3. 2008 integers are written on a piece of paper. It is known that the sum of any 100 numbers is positive. Show that the sum of all numbers is positive.

p4. Let p and q be prime numbers greater than 3. Prove that $p^2 - q^2$ is divisible by 24.

p5. Four villages A, B, C, and D are connected by trails as shown on the map.
https://cdn.artofproblemsolving.com/attachments/4/9/33ecc416792dacba65930caa61adbae09b829
png
On each path $A \to B \to C$ and $B \to C \to D$ there are 10 hills, on the path $A \to B \to D$ there are 22 hills, on the path $A \to D \to B$ there are 45 hills. A group of tourists starts from A and wants to reach D. They choose the path with the minimal number of hills. What is the best path for them?

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2013 **p1.** The clock is 2 hours 20 minutes ahead of the correct time each week. The clock is set to the correct time at midnight Sunday to Monday. What time does this clock show at 6pm correct time on Thursday?

p2. Five cities A, B, C, D, and E are located along the straight road in the alphabetical order. The sum of distances from B to A, C, D and E is 20 miles. The sum of distances from C to the other four cities is 18 miles. Find the distance between B and C.

p3. Does there exist distinct digits a, b, c, and d such that $\overline{abc} + \overline{c} = \overline{bda}$?
Here $\overline{abc}$ means the three digit number with digits a, b, and c.

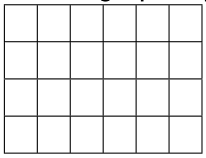
p4. Kuzya, Fyokla, Dunya, and Senya participated in a mathematical competition. Kuzya solved 8 problems, more than anybody else. Senya solved 5 problem, less than anybody else. Each problem was solved by exactly 3 participants. How many problems were there?

p5. Mr Mouse got to the cellar where he noticed three heads of cheese weighing 50 grams, 80 grams, and 120 grams. Mr. Mouse is allowed to cut simultaneously 10 grams from any two of the heads and eat them. He can repeat this procedure as many times as he wants. Can he make the weights of all three pieces equal?

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

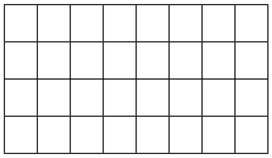
2014 **p1.** Find any integer solution of the puzzle: $WE + ST + RO + NG = 128$
(different letters mean different digits between 1 and 9).

p2. A 5×6 rectangle is drawn on the piece of graph paper (see the figure below). The side of each square on the graph paper is 1 cm long. Cut the rectangle along the sides of the graph squares in two parts whose areas are equal but perimeters are different by 2 cm.



p3. Three runners started simultaneously on a 1 km long track. Each of them runs the whole distance at a constant speed. Runner A is the fastest. When he runs 400 meters then the total distance run by runners B and C together is 680 meters. What is the total combined distance remaining for runners B and C when runner A has 100 meters left?

p4. There are three people in a room. Each person is either a knight who always tells the truth or a liar who always tells lies. The first person said ”We are all liars”. The second replied ”Only you are a liar”. Is the third person a liar or a knight?

p5. A 5×8 rectangle is divided into forty 1×1 square boxes (see the figure below). Choose 24 such boxes and one diagonal in each chosen box so that these diagonals don’t have common points. 

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2015 **p1.** To every face of a given cube a new cube of the same size is glued. The resulting solid has how many faces?

p2. A father and his son returned from a fishing trip. To make their catches equal the father gave to his son some of his fish. If, instead, the son had given his father the same number of fish, then father would have had twice as many fish as his son. What percent more is the father's catch more than his son's?

p3. A radio transmitter has 4 buttons. Each button controls its own switch: if the switch is OFF the button turns it ON and vice versa. The initial state of switches in unknown. The transmitter sends a signal if at least 3 switches are ON. What is the minimal number of times you have to push the button to guarantee the signal is sent?

p4. 19 matches are placed on a table to show the incorrect equation: $XXX + XIV = XV$. Move exactly one match to change this into a correct equation.

p5. Cut the grid shown into two parts of equal area by cutting along the lines of the grid.
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png

p6. A family of funny dwarfs consists of a dad, a mom, and a child. Their names are: A, R, and C (not in order). During lunch, C made the statements: "R and A have different genders" and "R and A are my parents", and A made the statements "I am C's dad" and "I am R's daughter." In fact, each dwarf told truth once and told a lie once. What is the name of the dad, what is the name of the child, and is the child a son or a daughter?

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2017 **p1.** Replace $*$'s by an arithmetic operations (addition, subtraction, multiplication or division) to obtain true equality

$$2 * 0 * 1 * 6 * 7 = 1.$$

p2. The interval of length 88 cm is divided into three unequal parts. The distance between middle points of the left and right parts is 46 cm. Find the length of the middle part.

p3. A 5×6 rectangle is drawn on a square grid. Paint some cells of the rectangle in such a way that every 3×2 sub-rectangle has exactly two cells painted.

p4. There are 8 similar coins. 5 of them are counterfeit. A detector can analyze any set of coins and show if there are counterfeit coins in this set. The detector neither determines which coins nare counterfeit nor how many counterfeit coins are there. How to run the detector twice to find for sure at least one counterfeit coin?

p5. There is a set of 20 weights of masses $1, 2, 3, ...$ and 20 grams. Can one divide this set into three groups of equal total masses?

p6. Replace letters A, B, C, D, E, F, G by the digits $0, 1, ..., 9$ to get true equality $AB + CD = EF * EG$ (different letters correspond to different digits, same letter means the same digit, AB, CD, EF, and EG are two-digit numbers).

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2018 **p1.** A Slavic dragon has three heads. A knight fights the dragon. If the knight cuts off one dragon's head three new heads immediately grow. Is it possible that the dragon has 2018 heads at some moment of the fight?

p2. Peter has two squares 3×3 and 4×4. He must cut one of them or both of them in no more than four parts in total. Is Peter able to assemble a square using all these parts?

p3. Usually, dad picks up Constantine after his music lessons and they drive home. However, today the lessons have ended earlier and Constantine started walking home. He met his dad 14 minutes later and they drove home together. They arrived home 6 minutes earlier than usually. Home many minutes earlier than usual have the lessons ended? Please, explain your answer.

p4. All positive integers from 1 to 2018 are written on a blackboard. First, Peter erased all numbers divisible by 7. Then, Natalie erased all remaining numbers divisible by 11. How many numbers did Natalie remove? Please, explain your answer.

p5. 30 students took part in a mathematical competition consisting of four problems. 25 students solved the first problem, 24 students solved the second problem, 22 students solved the third, and, finally, 21 students solved the fourth. Show that there are at least two students who solved all four problems.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2019 **p1.** It takes 12 months for Santa Claus to pack gifts. It would take 20 months for his apprentice to do the job. If they work together, how long will it take for them to pack the gifts?

p2. All passengers on a bus sit in pairs. Exactly $2/5$ of all men sit with women, exactly $2/3$ of all women sit with men. What part of passengers are men?

p3. There are 100 colored balls in a box. Every 10-tuple of balls contains at least two balls of the same color. Show that there are at least 12 balls of the same color in the box.

p4. There are 81 wheels in storage marked by their two types, say first and second type. Wheels of the same type weigh equally. Any wheel of the second type is much lighter than a wheel of the first type. It is known that exactly one wheel is marked incorrectly. Show that one can determine which wheel is incorrectly marked with four measurements.

p5. Remove from the figure below the specified number of matches so that there are exactly 5 squares of equal size left:
(a) 8 matches
(b) 4 matches
`https://cdn.artofproblemsolving.com/attachments/4/b/0c5a65f2d9b72fbea50df12e328c024a0c788`
`png`

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).

2022 **p1.** An animal farm has geese and pigs with a total of 30 heads and 84 legs. Find the number of pigs and geese on this farm.

p2. What is the maximum number of 1×1 squares of a 7×7 board that can be colored black in such a way that the black squares don't touch each other even at their corners? Show your answer on the figure below and explain why it is not possible to get more black squares satisfying the given conditions.
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png

p3. Decide whether it is possible to divide a regular hexagon into three equal not necessarily regular hexagons? A regular hexagon is a hexagon with equal sides and equal angles.
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png

p4. A rectangle is subdivided into a number of smaller rectangles. One observes that perimeters of all smaller rectangles are whole numbers. Is it possible that the perimeter of the original rectangle is not a whole number?

p5. Place parentheses on the left hand side of the following equality to make it correct.

$$4 \times 12 + 18 : 6 + 3 = 50$$

p6. Is it possible to cut a 16×9 rectangle into two equal parts which can be assembled into a square?

PS. You should use hide for answers. Collected here (https://artofproblemsolving.com/community/c5h2760506p24143309).

2023 **p1.** Solve: $INK + INK + INK + INK + INK + INK = PEN$
(INK and PEN are 3-digit numbers, and different letters stand for different digits).

p2. Two people play a game. They put 3 piles of matches on the table:
the first one contains 1 match, the second one 3 matches, and the third one 4 matches. Then they take turns making moves. In a move, a player may take any nonzero number of matches FROM ONE PILE. The player who takes the last match from the table loses the game.
a) The player who makes the first move can win the game. What is the winning first move?
b) How can he win? (Describe his strategy.)

p3. The planet Naboo is under attack by the imperial forces. Three rebellion camps are located at the vertices of a triangle. The roads connecting the camps are along the sides of the triangle. The length of the first road is less than or equal to 20 miles, the length of the second road is less than or equal to 30 miles, and the length of the third road is less than or equal to 45 miles. The Rebels have to cover the area of this triangle with a defensive field. What is the maximal area that they may need to cover?

p4. Money in Wonderland comes in $\$5$ and $\$7$ bills. What is the smallest amount of money you need to buy a slice of pizza that costs $\$1$ and get back your change in full? (The pizza man has plenty of $\$5$ and $\$7$ bills.) For example, having $\$7$ won't do, since the pizza man can only give you $\$5$ back.

p5. (a) Put 5 points on the plane so that each 3 of them are vertices of an isosceles triangle (i.e., a triangle with two equal sides), and no three points lie on the same line.
(b) Do the same with 6 points.

PS. You should use hide for answers. Collected here (`https://artofproblemsolving.com/community/c5h2760506p24143309`).