Problems from the 2022 BAMO-12 and BAMO-8 exams

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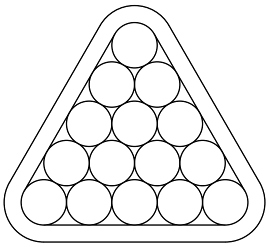
A If I have 100 cards with all the numbers 1 through 100 on them, how should I put them in order to create the largest possible number?

B You are bargaining with a salesperson for the price of an item. Your first offer is a dollars and theirs is b dollars. After you raise your offer by a certain percentage and they lower their offer by the same percentage, you arrive at an agreed price. What is that price, in terms of a and b?

C/1 The game of pool includes 15 balls that fit within a triangular rack as shown:



Seven of the balls are ”striped” (not colored with a single color) and eight are ”solid” (colored with a single color). Prove that no matter how the 15 balls are arranged in the rack, there must always be a pair of striped balls adjacent to each other.

D/2 Suppose that $p, p+d, p+2d, p+3d, p+4d$, and $p+5d$ are six prime numbers, where p and d are positive integers. Show that d must be divisible by $2, 3$, and 5.

E/3 A polygon is called *convex* if all its internal angles are smaller than 180°. Given a convex polygon, prove that one can find three distinct vertices A, P, and Q, where PQ is a side of the polygon, such that the perpendicular from A to the line PQ meets the segment PQ (possible at P of Q).

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4 Ten birds land on a 10-meter-long wire, each at a random point chosen uniformly along the wire. (That is, if we pick out any x-meter portion of the wire, there is an $\frac{x}{10}$ probability that a given bird will land there.) What is the probability that every bird sits more than one meter away from its closest neighbor?

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5 Sofiya and Marquis are playing a game. Sofiya announces to Marquis that she's thinking of a polynomial of the form $f(x) = x^3 + px + q$ with three integer roots that are not necessarily distinct. She also explains that all of the integer roots have absolute value less than (and not equal to) N, where N is some fixed number which she tells Marquis. As a ”move” in this game, Marquis can ask Sofiya about any number x and Sofiya will tell him whether $f(x)$ is positive negative, or zero. Marquis's goal is to figure out Sofiya's polynomial.

If $N = 3 \cdot 2^k$ for some positive integer k, prove that there is a strategy which allows Marquis to identify the polynomial after making at most $2k + 1$ ”moves”.