geometry problems from Team, Individual and General Rounds from Berkeley Math Tournament for High School

www.artofproblemsolving.com/community/c3006130

by parmenides51, IsabeltheCat, somepersonoverhere, mathisawesome

– Team Round

2012.3 Let ABC be a triangle with side lengths $AB = 2011$, $BC = 2012$, $AC = 2013$. Create squares $S_1 = ABB'A''$, $S_2 = ACC''A'$, and $S_3 = CBB''C'$ using the sides AB, AC, BC respectively, so that the side $B'A''$ is on the opposite side of AB from C, and so forth. Let square S_4 have side length $A''A'$, square S_5 have side length $C''C'$, and square S_6 have side length $B''B'$. Let $A(S_i)$ be the area of square S_i . Compute $\frac{A(S_4)+A(S_5)+A(S_6)}{A(S_1)+A(S_2)+A(S_3)}$?

2012.6 A circle with diameter AB is drawn, and the point P is chosen on segment AB so that $\frac{AP}{AB} = \frac{1}{42}$. Two new circles a and b are drawn with diameters AP and PB respectively. The perpendicular line to AB passing through P intersects the circle twice at points S and T . Two more circles s and t are drawn with diameters SP and ST respectively. For any circle ω let $A(\omega)$ denote the area of the circle. What is $\frac{A(s)+A(t)}{A(a)+A(b)}$?

2013.5 Circle C_1 has center O and radius OA, and circle C_2 has diameter OA. AB is a chord of circle C_1 and BD may be constructed with D on OA such that BD and OA are perpendicular. Let C be the point where C_2 and BD intersect. If $AC = 1$, find AB.

2013.8 A parabola has focus F and vertex V , where $VF = 10$. Let AB be a chord of length 100 that passes through F. Determine the area of $\triangle VAB$.

2014.4 In a right triangle, the altitude from a vertex to the hypotenuse splits the hypotenuse into two segments of lengths a and b. If the right triangle has area T and is inscribed in a circle of area C, find ab in terms of T and C.

2014.7 Let $VWXYZ$ be a square pyramid with vertex V with height 1, and with the unit square as its base. Let $STANFURD$ be a cube, such that face $FURD$ lies in the same plane as and shares the same center as square face $WXYZ$. Furthermore, all sides of $FURD$ are parallel to the sides of $WXYZ$. Cube $STANFURD$ has side length s such that the volume that lies inside the cube but outside the square pyramid is equal to the volume that lies inside the square pyramid but outside the cube. What is the value of s?

2014.13 Let ABC be a triangle with $AB = 16$, $AC = 10$, $BC = 18$. Let D be a point on AB such that $4AD = AB$ and let E be the foot of the angle bisector from B onto AC. Let P be the intersection of CD and BE. Find the area of the quadrilateral $ADPE$.

2015.4 Triangle ABC has side lengths $AB = 3$, $BC = 4$, and $CD = 5$. Draw line ℓ_A such that ℓ_A is parallel to BC and splits the triangle into two polygons of equal area. Define lines ℓ_B and ℓ_C analogously. The intersection points of ℓ_A, ℓ_B, and ℓ_C form a triangle. Determine its area.

2015.7 $X_1, X_2, ..., X_{2015}$ are 2015 points in the plane such that for all $1 \leq i, j \leq 2015$, the line segment $X_iX_{i+1} = X_jX_{j+1}$ and angle $\angle X_iX_{i+1}X_{i+2} = \angle X_jX_{j+1}X_{j+2}$ (with cyclic indices such that $X_{2016} = X_1$ and $X_{2017} = X_2$). Given fixed X_1 and X_2, determine the number of possible locations for X_3.

2015.9 Find the side length of the largest square that can be inscribed in the unit cube.

2015.16 Five points A, B, C, D, and E in three-dimensional Euclidean space have the property that $AB = BC = CD = DE = EA = 1$ and $\angle ABC = \angle BCD = \angle CDE = \angle DEA = 90^o$. Find all possible $\cos(\angle EAB)$.

2016.2 Jennifer wants to do origami, and she has a square of side length 1. However, she would prefer to use a regular octagon for her origami, so she decides to cut the four corners of the square to get a regular octagon. Once she does so, what will be the side length of the octagon Jennifer obtains?

2016.5 Let ABC be a right triangle with $AB = BC = 2$. Let ACD be a right triangle with angle $\angle DAC = 30$ degrees and $\angle DCA = 60$ degrees. Given that ABC and ACD do not overlap, what is the area of triangle BCD?

2016.10 What is the smallest possible perimeter of a triangle with integer coordinate vertices, area $\frac{1}{2}$, and no side parallel to an axis?

2016.11 Circles C_1 and C_2 intersect at points X and Y . Point A is a point on C_1 such that the tangent line with respect to C_1 passing through A intersects C_2 at B and C, with A closer to B than C, such that $2016 \cdot AB = BC$. Line XY intersects line AC at D. If circles C_1 and C_2 have radii of 20 and 16, respectively, find $\sqrt{1 + BC/BD}$.

2016.12 Consider a solid hemisphere of radius 1. Find the distance from its center of mass to the base.

2017.4 2 darts are thrown randomly at a circular board with center O, such that each dart has an equal probability of hitting any point on the board. The points at which they land are marked A and B. What is the probability that $\angle AOB$ is acute?

2017.6 The center of a square of side length 1 is placed uniformly at random inside a circle of radius 1. Given that we are allowed to rotate the square about its center, what is the probability that the entire square is contained within the circle for some orientation of the square?

2017.9 Let $AB = 10$ be a diameter of circle P. Pick point C on the circle such that $AC = 8$. Let the circle with center O be the incircle of $\triangle ABC$. Extend line AO to intersect circle P again at D. Find the length of BD.

2017.13 4 equilateral triangles of side length 1 are drawn on the interior of a unit square, each one of which shares a side with one of the 4 sides of the unit square. What is the common area enclosed by all 4 equilateral triangles?

2017.15 In triangle ABC, the angle at C is 30^o, side BC has length 4, and side AC has length 5. Let P be the point such that triangle ABP is equilateral and non-overlapping with triangle ABC. Find the distance from C to P.

2018.1 A circle with radius 5 is inscribed in a right triangle with hypotenuse 34 as shown below. What is the area of the triangle? Note that the diagram is not to scale.

2018.9 Circles A, B, and C are externally tangent circles. Line PQ is drawn such that PQ is tangent to A at P, tangent to B at Q, and does not intersect with C. Circle D is drawn such that it passes through the centers of A, B, and C. Let R be the point on D furthest from PQ. If A, B, and C have radii 3, 2, and 1, respectively, the area of triangle PQR can be expressed in the form of $a+b\sqrt{c}$, where a, b, and c are integers with c not divisible by any prime square. What is $a+b+c$?

2018.10 A rectangular prism has three distinct faces of area 24, 30, and 32. The diagonals of each distinct face of the prism form sides of a triangle. What is the triangle's area?

2018.15 Let triangle ABC have side lengths $AB = 13$, $BC = 14$, $AC = 15$. Let I be the incenter of ABC. The circle centered at A of radius AI intersects the circumcircle of ABC at H and J. Let L be a point that lies on both the incircle of ABC and line HJ. If the minimal possible value of AL is $\sqrt{n}$, where $n \in \mathbb{Z}$, find n.

2019.3 A cylinder with radius 5 and height 1 is rolling on the (unslanted) floor. Inside the cylinder, there is water that has constant height $\frac{15}{2}$ as the cylinder rolls on the floor. What is the volume of the water?

2019.13 Triangle $\triangle ABC$ has $AB = 13$, $BC = 14$, and $CA = 15$. $\triangle ABC$ has incircle γ and circumcircle ω. γ has center at I. Line AI is extended to hit ω at P. What is the area of quadrilateral $ABPC$?

2019.14 A regular hexagon has positive integer side length. A laser is emitted from one of the hexagon's corners, and is reflected off the edges of the hexagon until it hits another corner. Let a be the distance that the laser travels. What is the smallest possible value of a^2 such that $a > 2019$?

You need not simplify/compute exponents.

2020.3 An ant is at one corner of a unit cube. If the ant must travel on the box's surface, the shortest distance the ant must crawl to reach the opposite corner of the cube can be written in the form

$\sqrt{a}$, where a is a positive integer. Compute a.

2020.8 Let $ABCD$ be a unit square and let E and F be points inside $ABCD$ such that the line containing $\overline{EF}$ is parallel to $\overline{AB}$. Point E is closer to $\overline{AD}$ than point F is to $\overline{AD}$. The line containing $\overline{EF}$ also bisects the square into two rectangles of equal area. Suppose $[AEFB] = [DEFC] = 2[AED] = 2[BFC]$. The length of segment $\overline{EF}$ can be expressed as m/n , where m and n are relatively prime positive integers. Compute $m + n$.

2020.12 A hollow box (with negligible thickness) shaped like a rectangular prism has a volume of 108 cubic units. The top of the box is removed, exposing the faces on the inside of the box. What is the minimum possible value for the sum of the areas of the faces on the outside and inside of the box?

2020.14 In the star shaped figure below, if all side lengths are equal to 3 and the three largest angles of the figure are 210 degrees, its area can be expressed as $\frac{a\sqrt{b}}{c}$, where a, b, and c are positive integers such that a and c are relatively prime and that b is square-free. Compute $a + b + c$.
`https://cdn.artofproblemsolving.com/attachments/a/f/d16a78317b0298d6894c6bd62fbcd1a58943`
`png`

2020.16 Let T be the answer to question 18. Rectangle $ZOMR$ has $ZO = 2T$ and $ZR = T$. Point B lies on segment ZO, O' lies on segment OM, and E lies on segment RM such that $BR = BE = EO'$, and $\angle BEO' = 90^o$. Compute $2(ZO + O'M + ER)$.

PS. You had better calculate it in terms of T.

2020.20 Non-degenerate quadrilateral $ABCD$ with $AB = AD$ and $BC = CD$ has integer side lengths, and $\angle ABC = \angle BCD = \angle CDA$. If $AB = 3$ and $B \neq D$, how many possible lengths are there for BC?

2020.21 Let $\triangle ABC$ be a right triangle with legs $AB = 6$ and $AC = 8$. Let I be the incenter of $\triangle ABC$ and X be the other intersection of AI with the circumcircle of $\triangle ABC$. Find $\overline{AI} \cdot \overline{IX}$.

– Individual Round

2012.6 Let ABCD be a cyclic quadrilateral, with AB $= 7$, BC $= 11$, CD $= 13$, and DA $= 17$. Let the incircle of ABD hit BD at R and the incircle of CBD hit BD at S. What is RS?

2012.10 You are at one vertex of a equilateral triangle with side length 1. All of the edges of the equilateral triangle will reflect the laser beam perfectly (angle of incidence is equal to angle of reflection). Given that the laser beam bounces off exactly 137 edges and returns to the original vertex without touching any other vertices, let M be the maximum possible distance the beam could have traveled, and m be the minimum possible distance the beam could have traveled. Find $M^2 - m^2$.

2013.3 S-Corporation designs its logo by linking together 4 semicircles along the diameter of a unit circle. Find the perimeter of the shaded portion of the logo.
https://cdn.artofproblemsolving.com/attachments/8/6/f0eabd46f5f3a5806d49012b2f871a453b9e7
png

2013.4 Let $ABCD$ be a square with side length 2, and let a semicircle with flat side CD be drawn inside the square. Of the remaining area inside the square outside the semi-circle, the largest circle is drawn. What is the radius of this circle?

2013.6 Bubble Boy and Bubble Girl live in bubbles of unit radii centered at $(20, 13)$ and $(0, 10)$ respectively. Because Bubble Boy loves Bubble Girl, he wants to reach her as quickly as possible, but he needs to bring a gift; luckily, there are plenty of gifts along the x-axis. Assuming that Bubble Girl remains stationary, find the length of the shortest path Bubble Boy can take to visit the x-axis and then reach Bubble Girl (the bubble is a solid boundary, and anything the bubble can touch, Bubble Boy can touch too)

2013.12 Triangle ABC satisfies the property that $\angle A = a \log x$, $\angle B = a \log 2x$, and $\angle C = a \log 4x$ radians, for some real numbers a and x. If the altitude to side AB has length 8 and the altitude to side BC has length 9, find the area of $\triangle ABC$.

2013.14 Triangle ABC has incircle O that is tangent to AC at D. Let M be the midpoint of AC. E lies on BC so that line AE is perpendicular to BO extended. If $AC = 2013$, $AB = 2014$, $DM = 249$, find CE.

2013.15 Let $ABCD$ be a convex quadrilateral with $\angle ABD = \angle BCD$, $AD = 1000$, $BD = 2000$, $BC = 2001$, and $DC = 1999$. Point E is chosen on segment DB such that $\angle ABD = \angle ECD$. Find AE.

2013.19 Equilateral triangle ABC is inscribed in a circle. Chord AD meets BC at E. If $DE = 2013$, how many scenarios exist such that both DB and DC are integers (two scenarios are different if AB is different or AD is different)?

2014.2 Suppose $\triangle ABC$ is similar to $\triangle DEF$, with A, B, and C corresponding to D, E, and F respectively. If $\overline{AB} = \overline{EF}$, $\overline{BC} = \overline{FD}$, and $\overline{CA} = \overline{DE} = 2$, determine the area of $\triangle ABC$.

2014.8 Line segment AB has length 4 and midpoint M. Let circle C_1 have diameter AB, and let circle C_2 have diameter AM. Suppose a tangent of circle C_2 goes through point B to intersect circle C_1 at N. Determine the area of triangle AMN.

2014.10 A plane intersects a sphere of radius 10 such that the distance from the center of the sphere to the plane is 9. The plane moves toward the center of the bubble at such a rate that the increase in the area of the intersection of the plane and sphere is constant, and it stops once it reaches the center of the circle. Determine the distance from the center of the sphere to the plane after two-thirds of the time has passed.

2014.11 Suppose x, y, and 1 are side lengths of a triangle$T$ such that $x < 1$ and $y < 1$. Given x and y are chosen uniformly at random from all possible pairs (x, y), determine the probability that T is obtuse.

2014.12 Suppose four coplanar points A, B, C, and D satisfy $AB = 3$, $BC = 4$, $CA = 5$, and $BD = 6$. Determine the maximal possible area of $\triangle\ ACD$.

2014.13 A cylinder is inscribed within a sphere of radius 10 such that its volume is *almost-half* that of the sphere. If *almost-half* is defined such that the cylinder has volume $\frac{1}{2} + \frac{1}{250}$ times the sphere's volume, find the sum of all possible heights for the cylinder.

2014.17 A convex solid is formed in four-dimensional Euclidean space with vertices at the 24 possible permutations of $\{1, 2, 3, 4\}$ (so $(1, 2, 3, 4)$, $(1, 2, 4, 3)$, etc.). What is the product of the number of faces and edges of this solid?

2014.20 Suppose three circles of radius 5 intersect at a common point. If the three (other) pairwise intersections between the circles form a triangle of area 8, find the radius of the smallest possible circle containing all three circles.

2015.3 A quadrilateral $ABCD$ has a right angle at $\angle ABC$ and satisfies $AB = 12$, $BC = 9$, $CD = 20$, and $DA = 25$. Determine BD^2.
.

2015.7 In $\triangle\ ABC$, $\angle B = 46^o$ and $\angle C = 48^o$. A circle is inscribed in $\triangle\ ABC$ and the points of tangency are connected to form PQR. What is the measure of the largest angle in $\triangle\ PQR$?

2015.17 A circle intersects square $ABCD$ at points A, E, and F, where E lies on AB and F lies on AD, such that $AE + AF = 2(BE + DF)$. If the square and the circle each have area 1, determine the area of the union of the circle and square.

2015.T1 Compute the surface area of a rectangular prism with side lengths $2, 3, 4$.

2016.3 Consider an equilateral triangle and square, both with area 1. What is the product of their perimeters?

2016.4 Let ABC have side lengths 3, 4, and 5. Let P be a point inside ABC. What is the minimum sum of lengths of the altitudes from P to the side lengths of ABC?

2016.14 Three circles of radius 1 are inscribed in a square of side length s, such that the circles do not overlap or coincide with each other. What is the minimum s where such a configuration is possible?

2016.16 What is the radius of the largest sphere that fits inside the tetrahedron whose vertices are the points $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$?

2016.17 Consider triangle ABC in xy-plane where A is at the origin, B lies on the positive x-axis, C is on the upper right quadrant, and $\angle A = 30^o$, $\angle B = 60^o$,$\angle C = 90^o$. Let the length $BC = 1$. Draw the angle bisector of angle $\angle C$, and let this intersect the y-axis at D. What is the area of quadrilateral $ADBC$?

2016.19 Regular tetrahedron $P_1P_2P_3P_4$ has side length 1. Define P_i for $i > 4$ to be the centroid of tetrahedron $P_{i-1}P_{i-2}P_{i-3}P_{i-4}$, and $P_\infty = \lim_{n\to\infty} P_n$. What is the length of P_5P_∞?

2017.2 Barack is an equilateral triangle and Michelle is a square. If Barack and Michelle each have perimeter 12, find the area of the polygon with larger area.

2017.10 Let S be the set of points A in the Cartesian plane such that the four points A, $(2, 3)$, $(-1, 0)$, and $(0, 6)$ form the vertices of a parallelogram. Let P be the convex polygon whose vertices are the points in S. What is the area of P?

2017.13 Two points are located 10 units apart, and a circle is drawn with radius r centered at one of the points. A tangent line to the circle is drawn from the other point. What value of r maximizes the area of the triangle formed by the two points and the point of tangency?

2017.16 Let ABC be a triangle with $AB = 3$, $BC = 5$, $AC = 7$, and let P be a point in its interior. If G_A, G_B, G_C are the centroids of $\triangle\ PBC$, $\triangle\ PAC$, $\triangle\ PAB$, respectively, find the maximum possible area of $\triangle\ G_AG_BG_C$.

2017.17 Triangle ABC is drawn such that $\angle A = 80^o$, $\angle B = 60^o$, and $\angle C = 40^o$. Let the circumcenter of $\triangle\ ABC$ be O, and let ω be the circle with diameter AO. Circle ω intersects side AC at point P. Let M be the midpoint of side BC, and let the intersection of ω and PM be K. Find the measure of $\angle MOK$.

2017.19 Let T be the triangle in the xy-plane with vertices $(0, 0)$, $(3, 0)$, and $\left(0, \frac{3}{2}\right)$. Let E be the ellipse inscribed in T which meets each side of T at its midpoint. Find the distance from the center of E to $(0, 0)$.

2019.2 Let A, B, C be unique collinear points$AB = BC = \frac{1}{3}$. Let P be a point that lies on the circle centered at B with radius $\frac{1}{3}$ and the circle centered at C with radius $\frac{1}{3}$. Find the measure of angle $\angle PAC$ in degrees.

2019.5 Point P is $\sqrt{3}$ units away from plane A. Let Q be a region of A such that every line through P that intersects A in Q intersects A at an angle between 30^o and 60^o . What is the largest possible area of Q?

2019.6 How many square inches of paint are needed to fully paint a regular 6-sided die with side length 2 inches, except for the $\frac{1}{3}$-inch diameter circular dots marking 1 through 6 (a different number per side)? The paint has negligible thickness, and the circular dots are non-overlapping.

2019.7 Let $\triangle ABC$ be an equilateral triangle with side length M such that points E_1 and E_2 lie on side AB, F_1 and F_2 lie on side BC, and $G1$ and $G2$ lie on side AC, such that

$$m = \overline{AE_1} = \overline{BE_2} = \overline{BF_1} = \overline{CF_2} = \overline{CG_1} = \overline{AG_2}$$

and the area of polygon $E_1E_2F_1F_2G_1G_2$ equals the combined areas of $\triangle AE_1G_2$, $\triangle BF_1E_2$, and $\triangle CG_1F_2$. Find the ratio $\frac{m}{M}$.
`https://cdn.artofproblemsolving.com/attachments/a/0/88b36c6550c42d913cdddd4486a3dde251327`
`png`

2019.10 Let $MATH$ be a square with $MA = 1$. Point B lies on AT such that $\angle MBT = 3.5\angle BMT$. What is the area of $\triangle BMT$?

2019.11 A regular 17-gon with vertices $V_1, V_2, ..., V_{17}$ and sides of length 3 has a point P on V_1V_2 such that $V_1P = 1$. A chord that stretches from V_1 to V_2 containing P is rotated within the interior of the heptadecagon around V_2 such that the chord now stretches from V_2 to V_3. The chord then hinges around V_3, then V_4, and so on, continuing until P is back at its original position. Find the total length traced by P.

2019.13 Two circles O_1 and O_2 intersect at points A and B. Lines $\overline{AC}$ and $\overline{BD}$ are drawn such that C is on O_1 and D is on O_2 and $\overline{AC} \perp \overline{AB}$ and $\overline{BD} \perp \overline{AB}$. If minor arc $AB = 45$ degrees relative to O_1 and minor arc $AB = 60$ degrees relative to O_2 and the radius of $O_2 = 10$, the area of quadrilateral $CADB$ can be expressed in simplest form as $a + b\sqrt{k} + c\sqrt{\ell}$. Compute $a + b + c + k + \ell$.

2019.16 Let ABC be a triangle with $AB = 26$, $BC = 51$, and $CA = 73$, and let O be an arbitrary point in the interior of $\triangle ABC$. Lines ℓ_1, ℓ_2, and ℓ_3 pass through O and are parallel to $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively. The intersections of ℓ_1, ℓ_2, and ℓ_3 and the sides of $\triangle ABC$ form a hexagon whose area is A. Compute the minimum value of A.

2019.17 Let C be a circle of radius 1 and O its center. Let $\overline{AB}$ be a chord of the circle and D a point on $\overline{AB}$ such that $OD = \frac{\sqrt{2}}{2}$ such that D is closer to A than it is to B, and if the perpendicular line at D with respect to $\overline{AB}$ intersects the circle at $E$and F, $AD = DE$. The area of the region of the circle enclosed by $\overline{AD}$, $\overline{DE}$, and the minor arc AE may be expressed as $\frac{a+b\sqrt{c}+d\pi}{e}$ where a, b, c, d, e are integers, gcd $(a, b, d, e) = 1$, and c is squarefree. Find $a + b + c + d + e$

2019.T4 Consider a regular triangular pyramid with base $\triangle ABC$ and apex D. If we have $AB = BC = AC = 6$ and $AD = BD = CD = 4$, calculate the surface area of the circumsphere of the pyramid.

2020.5 A Yule log is shaped like a right cylinder with height 10 and diameter 5. Freya cuts it parallel to its bases into 9 right cylindrical slices. After Freya cut it, the combined surface area of the slices of the Yule log increased by $a\pi$. Compute a.

2020.9 A circle C with radius 3 has an equilateral triangle inscribed in it. Let D be a circle lying outside the equilateral triangle, tangent to C, and tangent to the equilateral triangle at the midpoint of one of its sides. The radius of D can be written in the form m/n , where m and n are relatively prime positive integers. Compute $m + n$.

2020.11 Equilateral triangle ABC has side length 2. A semicircle is drawn with diameter BC such that it lies outside the triangle, and minor arc BC is drawn so that it is part of a circle centered at A. The area of the "lune" that is inside the semicircle but outside sector ABC can be expressed in the form $\sqrt{p} - \frac{q\pi}{r}$, where p, q, and r are positive integers such that q and r are relatively prime. Compute $p + q + r$.
https://cdn.artofproblemsolving.com/attachments/7/7/f349a807583a83f93ba413bebf07e0132655
png

2020.13 Sheila is making a regular-hexagon-shaped sign with side length 1. Let $ABCDEF$ be the regular hexagon, and let R, S, T and U be the midpoints of FA, BC, CD and EF, respectively. Sheila splits the hexagon into four regions of equal width: trapezoids $ABSR$, $RSCF$, $FCTU$, and $UTDE$. She then paints the middle two regions gold. The fraction of the total hexagon that is gold can be written in the form m/n , where m and n are relatively prime positive integers. Compute $m + n$.
https://services.artofproblemsolving.com/download.php?id=YXR0YWNobWVudHMvYS9lLzIwOTVmZmVi
=\&rn=MjAyMCBCTVQgSW5kaXZpZHVhbCAxMy5wbmc=

2020.16 The triangle with side lengths $3, 5$, and k has area 6 for two distinct values of k: x and y. Compute $|x^2 - y^2|$.

2020.19 Alice is standing on the circumference of a large circular room of radius 10. There is a circular pillar in the center of the room of radius 5 that blocks Alice's view. The total area in the room Alice can see can be expressed in the form $\frac{m\pi}{n} + p\sqrt{q}$, where m and n are relatively prime positive integers and p and q are integers such that q is square-free. Compute $m + n + p + q$. (Note that the pillar is not included in the total area of the room.)
https://cdn.artofproblemsolving.com/attachments/5/1/26e8aa6d12d9dd85bd5b284b6176870c7d11b
png

2020.23 Circle Γ has radius 10, center O, and diameter AB. Point C lies on Γ such that $AC = 12$. Let P be the circumcenter of $\triangle AOC$. Line AP intersects Γ at Q, where Q is different from A. Then the value of $\frac{AP}{AQ}$ can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

2020.T1 An *exterior* angle is the supplementary angle to an interior angle in a polygon. What is the sum of the exterior angles of a triangle and dodecagon (12-gon), in degrees?

2020.T3 $\triangle ABC$ has $AB = 5$, $BC = 12$, and $AC = 13$. A circle is inscribed in $\triangle ABC$, and MN tangent to the circle is drawn such that M is on $\overline{AC}$, N is on $\overline{BC}$, and $\overline{MN} \parallel \overline{AB}$. The area of $\triangle MNC$ is m/n , where m and $n$are relatively prime positive integers. Find $m + n$.

– General Round

2021.3 A scalene acute triangle has angles whose measures (in degrees) are whole numbers. What is the smallest possible measure of one of the angles, in degrees?

2021.6 A toilet paper roll is a cylinder of radius 8 and height 6 with a hole in the shape of a cylinder of radius 2 and the same height. That is, the bases of the roll are annuli with inner radius 2 and outer radius 8. Compute the surface area of the roll.

2021.10 Triangle $\triangle ABC$ has side lengths $AB = AC = 27$ and $BC = 18$. Point D is on $\overline{AB}$ and point E is on $\overline{AC}$ such that $\angle BCD = \angle CBE = \angle BAC$. Compute DE.

2021.23 Shivani has a single square with vertices labeled $ABCD$. She is able to perform the following transformations: • She does nothing to the square. • She rotates the square by 90, 180, or 270 degrees. • She reflects the square over one of its four lines of symmetry.
For the first three timesteps, Shivani only performs reflections or does nothing. Then for the next three timesteps, she only performs rotations or does nothing. She ends up back in the square's original configuration. Compute the number of distinct ways she could have achieved this.

2021.25 Let $\triangle BMT$ be a triangle with $BT = 1$ and height 1. Let O_0 be the centroid of $\triangle BMT$, and let $\overline{BO_0}$ and $\overline{TO_0}$ intersect $\overline{MT}$ and $\overline{BM}$ at B_1 and T_1, respectively. Similarly, let O_1 be the centroid of $\triangle B_1MT_1$, and in the same way, denote the centroid of $\triangle B_nMT_n$ by O_n, the intersection of $\overline{BO_n}$ with $\overline{MT}$ by B_{n+1}, and the intersection of $\overline{TO_n}$ with $\overline{BM}$ by T_{n+1}. Compute the area of quadrilateral $MBO_{2021}T$.

2021.T2 Compute the radius of the largest circle that fits entirely within a unit cube.

2021.T4 Let z_1, z_2, and z_3 be the complex roots of the equation $(2z - 3\overline{z})^3 = 54i + 54$. Compute the area of the triangle formed by z_1, z_2, and z_3 when plotted in the complex plane.

2022.4 Big Chungus has been thinking of a new symbol for BMT, and the drawing below is what he came up with. If each of the 16 small squares in the grid are unit squares, what is the area of the shaded region?
`https://cdn.artofproblemsolving.com/attachments/c/6/285948385f4644756f5d1fc50602af34e7530`
`png`

2022.12 Parallelograms $ABGF$, $CDGB$ and $EFGD$ are drawn so that $ABCDEF$ is a convex hexagon, as shown. If $\angle ABG = 53^o$ and $\angle CDG = 56^o$, what is the measure of $\angle EFG$, in degrees?
https://cdn.artofproblemsolving.com/attachments/9/f/79d163662e02bc40d2636a76b73f632e59d58
png