Finals 2015
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–	Day 1
1	In triangle ABC the angle $\angle A$ is the smallest. Points D, E lie on sides AB, AC so that $\angle CBE = \angle DCB = \angle BAC$. Prove that the midpoints of AB, AC, BE, CD lie on one circle.
2	Let P be a polynomial with real coefficients. Prove that if for some integer k $P(k)$ isn't integral, then there exist infinitely many integers m, for which $P(m)$ isn't integral.
3	Find the biggest natural number m that has the following property: among any five 500-element subsets of $\{1, 2, \dots, 1000\}$ there exist two sets, whose intersection contains at least m numbers.
–	Day 2
1	Solve the system $$\begin{cases} x+y+z=1 \\ x^5+y^5+z^5=1 \end{cases}$$ in real numbers.
2	Prove that diagonals of a convex quadrilateral are perpendicular if and only if inside of the quadrilateral there is a point, whose orthogonal projections on sides of the quadrilateral are vertices of a rectangle.
3	Prove that for each positive integer a there exists such an integer $b > a$, for which $1 + 2^a + 3^a$ divides $1 + 2^b + 3^b$.