Turkey EGMO TST 2014

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Day 1 February 13th

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1 Let D be the midpoint of the side BC of a triangle ABC and AD intersect the circumcircle of ABC for the second time at E. Let P be the point symmetric to the point E with respect to the point D and Q be the point of intersection of the lines CP and AB. Prove that if A, C, D, Q are concyclic, then the lines BP and AC are perpendicular.

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2 p is a prime. Find the all (m, n, p) positive integer triples satisfy $m^3 + 7p^2 = 2^n$.

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3 Denote by $d(n)$ be the biggest prime divisor of $|n| > 1$. Find all polynomials with integer coefficients satisfy;

$$P(n + d(n)) = n + d(P(n))$$

for the all $|n| > 1$ integers such that $P(n) > 1$ and $d(P(n))$ can be defined.

Day 2 February 14th

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4 Let x, y, z be positive real numbers such that $x + y + z \geq x^2 + y^2 + z^2$. Show that;

$$\frac{x^2+3}{x^3+1} + \frac{y^2+3}{y^3+1} + \frac{z^2+3}{z^3+1} \geq 6$$

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5 Let ABC be a triangle with circumcircle ω and let ω_A be a circle drawn outside ABC and tangent to side BC at A_1 and tangent to ω at A_2. Let the circles ω_B and ω_C and the points B_1, B_2, C_1, C_2 are defined similarly. Prove that if the lines AA_1, BB_1, CC_1 are concurrent, then the lines AA_2, BB_2, CC_2 are also concurrent.

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6 For a given integer $n \geq 3$, let $S_1, S_2, \ldots, S_m$ be distinct three-element subsets of the set $\{1, 2, \ldots, n\}$ such that for each $1 \leq i, j \leq m; i \neq j$ the sets $S_i \cap S_j$ contain exactly one element. Determine the maximal possible value of m for each n.
