Turkey EGMO TST 2022

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1 Given an acute angle triangle ABC with circumcircle Γ and circumcenter O. A point P is taken on the line BC but not on $[BC]$. Let K be the reflection of the second intersection of the line AP and Γ with respect to OP. If M is the intersection of the lines AK and OP, prove that $\angle OMB + \angle OMC = 180^\circ$.

2 We are given some three element subsets of $\{1, 2, \dots, n\}$ for which any two of them have at most one common element. We call a subset of $\{1, 2, \dots, n\}$ *nice* if it doesn't include any of the given subsets. If no matter how the three element subsets are selected in the beginning, we can add one more element to every 29-element *nice* subset while keeping it nice, find the minimum value of n.

3 Find all pairs of integers (a, b) satisfying the equation $a^7(a-1) = 19b(19b+2)$.

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4 On a table there are 100 red and k white buckets for which all of them are initially empty. In each move, a red and a white bucket is selected and an equal amount of water is added to both of them. After some number of moves, there is no empty bucket and for every pair of buckets that are selected together at least once during the moves, the amount of water in these buckets is the same. Find all the possible values of k.

5 We are given three points A, B, C on a semicircle. The tangent lines at A and B to the semicircle meet the extension of the diameter at points M, N respectively. The line passing through A that is perpendicular to the diameter meets NC at R, and the line passing through B that is perpendicular to the diameter meets MC at S. If the line RS meets the extension of the diameter at Z, prove that ZC is tangent to the semicircle.

6 Let x, y, z be positive real numbers satisfying the equations

$$xyz = 1 \text{ and } \frac{y}{z}(y-x^2) + \frac{z}{x}(z-y^2) + \frac{x}{y}(x-z^2) = 0$$

What is the minimum value of the ratio of the sum of the largest and smallest numbers among x, y, z to the median of them.