CJMO - Canadian Junior Mathematical Olympiad 2022

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1 Let ABC be an acute angled triangle with circumcircle Γ. The perpendicular from A to BC intersects Γ at D, and the perpendicular from B to AC intersects Γ at E. Prove that if $|AB| = |DE|$, then $\angle ACB = 60^o$.
the official wording

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2 You have an infinite stack of T-shaped tetrominoes (composed of four squares of side length 1), and an n × n board. You are allowed to place some tetrominoes on the board, possibly rotated, as long as no two tetrominoes overlap and no tetrominoes extend off the board. For which values of n can you cover the entire board?

– those were also the first CMO problems

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3 Assume that real numbers a and b satisfy

$$ab + \sqrt{ab+1} + \sqrt{a^2+b}\sqrt{a+b^2} = 0.$$

Find, with proof, the value of

$$b\sqrt{a^2+b} + a\sqrt{b^2+a}.$$

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4 Let $d(k)$ denote the number of positive integer divisors of k. For example, $d(6) = 4$ since 6 has 4 positive divisors, namely, $1, 2, 3$, and 6. Prove that for all positive integers n,

$$d(1) + d(3) + d(5) + ... + d(2n-1) \leq d(2) + d(4) + d(6) + ... + d(2n).$$

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5 Vishal starts with n copies of the number 1 written on the board. Every minute, he takes two numbers a, b and replaces them with either $a+b$ or $\min(a^2, b^2)$. After $n-1$ there is 1 number on the board. Let the maximal possible value of this number be $f(n)$. Prove $2^{n/3} < f(n) \leq 3^{n/3}$.
