**Basically containing secondary national question (for now.)**

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**–** Secondary

**1** Find all solutions for real $x$,

$$\lfloor x \rfloor^3 - 7\lfloor x + \frac{1}{3} \rfloor = -13.$$

**2** In $\triangle ABC, \angle BAC$ is a right angle. $BP$ and $CQ$ are bisectors of $\angle B$ and $\angle C$ respectively, which intersect $AC$ and $AB$ at $P$ and $Q$ respectively. Two perpendicular segments $PM$ and $QN$ are drawn on $BC$ from $P$ and $Q$ respectively. Find the value of $\angle MAN$ with proof.

**3** Prove that if the numbers $3, 4, 5, \dots, 3^5$ are partitioned into two disjoint sets, then in one of the sets the number $a, b, c$ can be found such that $ab = c$. ($a, b, c$ may not be pairwise distinct)

**4** Pratyya and Payel have a number each, $n$ and $m$ respectively, where $n > m$. Everyday, Pratyya multiplies his number by $2$ and then subtracts $2$ from it, and Payel multiplies his number by $2$ and then add $2$ to it. In other words, on the first day their numbers will be $(2n - 2)$ and $(2m + 2)$ respectively. Find minimum integer $x$ with proof such that if $n - m \geq x$, then Pratyya's number will be larger than Payel's number everyday.

**5** In an acute triangle $\triangle ABC$, the midpoint of $BC$ is $M$. Perpendicular lines $BE$ and $CF$ are drawn respectively on $AC$ from $B$ and on $AB$ from $C$ such that $E$ and $F$ lie on $AC$ and $AB$ respectively. The midpoint of $EF$ is $N$. $MN$ intersects $AB$ at $K$. Prove that, the four points $B, K, E, M$ lie on the same circle.

**6** About $5$ years ago, Joydip was researching on the number $2017$. He understood that $2017$ is a prime number. Then he took two integers $a, b$ such that $0 < a, b < 2017$ and $a + b \neq 2017$. He created two sequences $A_1, A_2, \dots, A_{2016}$ and $B_1, B_2, \dots, B_{2016}$ where $A_k$ is the remainder upon dividing $ak$ by $2017$, and $B_k$ is the remainder upon dividing $bk$ by $2017$. Among the numbers $A_1 + B_1, A_2 + B_2, \dots A_{2016} + B_{2016}$ count of those that are greater than $2017$ is $N$. Prove that $N = 1008$.

**7** Sabbir noticed one day that everyone in the city of BdMO has a distinct word of length $10$, where each letter is either $A$ or $B$. Sabbir saw that two citizens are friends if one of their words can be altered a few times using a special rule and transformed into the other ones word. The rule is, if somewhere in the word $ABB$ is located consecutively, then these letters can be changed to $BBA$ or if $BBA$ is located somewhere in the word consecutively, then these letters can be changed to $ABB$ (if wanted, the word can be kept as it is, without making this change.) For example $AABBA$ can be transformed into $AAABB$ (the opposite is also possible.) Now Sabbir made a team of $N$ citizens where no one is friends with anyone. What is the highest value of $N$.

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**8** Solve the following problems -

A) Find any $158$ consecutive integers such that the sum of digits for any of the numbers is not divisible by $17$.

B) Prove that, among any $159$ consecutive integers there will always be at least one integer whose sum of digits is divisible by $17$.

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