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– Grade 9

1 Given $a, b, c > 0$ such that

$$a+b+c+\frac{1}{abc}=\frac{19}{2}$$

What is the greatest value for a?

– Grade 10

3 Let (a_n) and (b_n) be sequences of real numbers, such that $a_1 = b_1 = 1$, $a_{n+1} = a_n + \sqrt{a_n}$, $b_{n+1} = b_n + \sqrt[3]{b_n}$ for all positive integers n. Prove that there is a positive integer n for which the inequality $a_n \le b_k < a_{n+1}$ holds for exactly 2021 values of k.

5 Find all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that

$$f(x)^2 = f(xy) + f(x + f(y)) - 1$$

for all $x, y \in \mathbb{R}^+$

6 See problem 5 of grade 11

– Grade 11

4 Given acute triangle ABC with circumcircle Γ and altitudes AD, BE, CF, line AD cuts Γ again at P and PF, PE meet Γ again at R, Q. Let O_1, O_2 be the circumcenters of $\triangle BFR$ and $\triangle CEQ$ respectively. Prove that O_1O_2 bisects $\overline{EF}$.

5 Let a be a positive integer. Prove that for any pair (x, y) of integer solutions of equation

$$x(y^2 - 2x^2) + x + y + a = 0$$

we have:

$$|x| \leqslant a + \sqrt{2a^2 + 2}$$