The problems from the CCA Math Bonanza held on 4/23/2022.

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by pooh_bear

–   Individual Round

I1   Asteroids A and B have circular orbits around the same star. Asteroid A is located 400 km away from the star and takes 8000 hours to complete one full revolution. Asteroid B is located 100 km away and the speed of Asteroid B is twice the speed of Asteroid A. Find how long it takes for Asteroid B to complete one full revolution in hours.

2022 CCA Math Bonanza Individual Round#1

I2   Alice, Bob, Cassie, Dave, and Ethan are going on a road trip and need to arrange themselves among a drivers seat, a passenger seat, and three distinguishable back row seats. Alice, Bob, and Cassie are not allowed to drive. Alice and Bob are also not allowed to sit in the front passenger seat. Find the number of possible seating arrangements.

2022 CCA Math Bonanza Individual Round#2

I3   Let $S = 1, 2, \cdots, 100$. X is a subset of S such that no two distinct elements in X multiply to an element in X. Find the maximum number of elements of X.

2022 CCA Math Bonanza Individual Round#3

I4   Burrito Bear has a white unit square. She inscribes a circle inside of the square and paints it black. She then inscribes a square inside the black circle and paints it white. Burrito repeats this process indefinitely. The total black area can be expressed as $\frac{a\pi+b}{c}$. Find $a + b + c$.

2022 CCA Math Bonanza Individual Round#4

I5   Let Γ_1 be a circle with radius $\frac{5}{2}$. A, B, and C are points on Γ_1 such that $\overline{AB} = 3$ and $\overline{AC} = 5$. Let Γ_2 be a circle such that Γ_2 is tangent to AB and BC at Q and R, and Γ_2 is also internally tangent to Γ_1 at P. Γ_2 intersects AC at X and Y. $[PXY]$ can be expressed as $\frac{a\sqrt{b}}{c}$. Find $a + b + c$.

2022 CCA Math Bonanza Individual Round#5

I6   Let regular tetrahedron $ABCD$ have center O. Find $\tan^2(\angle AOB)$.

2022 CCA Math Bonanza Individual Round#6

I7   Let

$$A = \{2, 4, \ldots, 1000\},$$
$$B = \{3, 6, \ldots, 999\},$$

$$C = \{5, 10, \ldots, 1000\},$$
$$D = \{7, 14, \ldots, 994\},$$
$$E = \{11, 22, \ldots, 990\},$$
$$\text{and } F = \{13, 26, \ldots, 988\}.$$

Find the number of elements in the set $(((((A \cup B) \cap C) \cup D) \cap E) \cup F)$.

2022 CCA Math Bonanza Individual Round#7

I8 Lason Jiu gives a problem to Sick Nong and Ayush Agrawal. Sick takes 6 minutes to solve the problem, while Ayush takes 9 minutes. Sick has a 1/3 chance of solving correctly and Ayush has a 2/3 chance of solving correctly. If they solved it incorrectly, they resume solving with the same time and accuracy. Lason gives a rubber chicken to the first person who solves it correctly. If Sick and Ayush solve the question at the same time, Lason checks Sick's work first. The probability that Ayush wins the rubber chicken can be expressed as $\frac{p}{q}$. Find $p + q$.

2022 CCA Math Bonanza Individual Round#8

I9 Find the maximum value of x such that x divides all $p^{32} - 1$ for all primes $p > 20$.

2022 CCA Math Bonanza Individual Round#9

I10 Let $\overline{AB}$ be a line segment of length 2, C_1 be the circle with diameter $\overline{AB}$, C_0 be the circle with radius 2 externally tangent to C_1 at A, and C_2 be the circle with radius 3 externally tangent to C_1 at B. Duck D_1 is located at point B, Duck D_2 is located on C_2, 270 degrees counterclockwise from B, and Duck D_0 is located on C_0, 270 degrees counterclockwise from A. At the same time, the ducks all start running counterclockwise around their corresponding circles, with each duck taking the same amount of time to complete a full lap around its circle. When the 3 ducks are first collinear, the distance between D_0 and D_2 can be expressed as $p\sqrt{q}$. Find $p + q$.

2022 CCA Math Bonanza Individual Round#10

I11 A river is bounded by the lines $x = 0$ and $x = 25$, with a current of 2 units/s in the positive y-direction. At $t = 0$, a mallard is at $(0, 0)$, and a wigeon is at $(25, 0)$. They start swimming with a constant speed such that they meet at $(x, 22)$. The mallard has a speed of 4 units/s relative to the water, and the wigeon has a speed of 3 units/s relative to the water. Find the value of x.

2022 CCA Math Bonanza Individual Round#11

I12 Find the number of 8-tuples of binary inputs $\{A, B, C, D, E, F, G, H\}$ such that

$$\{(A \text{ AND } B) \text{ OR } (C \text{ AND } D)\} \text{ AND } \{(E \text{ AND } F) \text{ OR } (G \text{ AND } H)\}$$

$$= \{(A \text{ OR } B) \text{ AND } (C \text{ OR } D)\} \text{ OR } \{(E \text{ OR } F) \text{ AND } (G \text{ OR } H)\}$$

The AND gates produce an output that is ON only if both the inputs are ON, and the OR gates produce an output that is OFF only if both inputs are OFF.

2022 CCA Math Bonanza Individual Round#12

I13   Let triangle A_1BC have sides $A_1B = 5$, $A_1C = 12$, and $BC = 13$. For all natural numbers i, let B_i be the foot of the altitude from A_i to BC, let A_{2i} be the foot of the altitude from B_i to A_1B, and let A_{2i+1} be the foot of the altitude from B_i to A_1C.

$$\sum_{i=1}^{7} A_iB_i = \frac{p}{q}$$

Find $p + q$.

2022 CCA Math Bonanza Individual Round#13

I14   Let ABC be a triangle with side lengths $AB = 6$, $AC = 7$, and $BC = 8$. Let H be the orthocenter of $\triangle ABC$ and H' be the reflection of H across the midpoint M of BC. $\frac{[ABH']}{[ACH']}$ can be expressed as $\frac{p}{q}$. Find $p + q$.

2022 CCA Math Bonanza Individual Round#14

I15   Let P, A, B, C, D be points on a plane such that $PA = 9$, $PB = 19$, $PC = 9$, $PD = 5$, $\angle APB = 120°$, $\angle BPC = 45°$, $\angle CPD = 60°$, and $\angle DPA = 135°$. Let G_1, G_2, G_3, and G_4 be the centroids of triangles PAB, PBC, PCD, PDA. $[G_1G_2G_3G_4]$ can be expressed as $a\sqrt{b}+c\sqrt{d}$. Find $a+b+c+d$.

2022 CCA Math Bonanza Individual Round#15

–   Team Round

T1   Let a, b, c, and d be positive integers such that $77^a \cdot 637^b = 143^c \cdot 49^d$. Compute the minimal value of $a + b + c + d$.

2022 CCA Math Bonanza Team Round#1

T2   CCA's B building has 6 rooms on the second floor, labeled B201 to B206, as well as 8 rooms on the first floor, labeled B101 to B108. Annie is currently in room B205. Each minute, she chooses to stay or change floors with equal probability, and chooses a classroom on that floor to go to at random (she can stay in the classroom that she's already in). B104, B108, and B203 are the only rooms that have teachers who will scold her for randomly walking around during class time. The probability that she is first scolded in room B203 can be expressed as $\frac{p}{q}$. Compute $p + q$.

2022 CCA Math Bonanza Team Round#2

T3   The smallest possible volume of a cylinder that will fit nine spheres of radius 1 can be expressed as $x\pi$ for some value of x. Compute x.

2022 CCA Math Bonanza Team Round#3

T4   Let there exist a configuration of *exactly* 1 black king, n black chess pieces (each of which can be a pawn, knight, bishop, rook, or queen), and a white *anti-king* on a standard 8x8 board in which the white *anti-king* is not under attack, but will be if it is moved. Compute the minimal value of n.

*An *anti-king* can move to any square is **not** 1 square vertically, horizontally, or diagonally. It can also capture undefended pieces.

2022 CCA Math Bonanza Team Round#4

T5   Maggie Waggie organizes a pile of 127 calculus tests in alphabetical order, with Joccy Woccy's test being 64th in the pile. While Maggie isn't looking, Joccy walks over and randomly scrambles the entire pile of tests. When Maggie returns, she is oblivious to the fact that Joccy has tampered with the list. She uses a binary search algorithm to find Joccy's test, where she looks at the test in the middle of the pile. If the test is not Joccy's, she binary searches the top half of the list if the test appears after Joccy's name when arranged alphabetically, or the bottom half of the list otherwise. The probability that Maggie finds Joccy's test can be expressed as $\frac{p}{q}$. Compute $p+q$.

2022 CCA Math Bonanza Team Round#5

T6   A bird starts with 300 ml of blood at 100 degrees in its body, 50 ml of blood at 0 degrees in its feet. Every minute, 50 ml of blood flows from the body to the feet, and 50 ml of blood at 40

2022 CCA Math Bonanza Team Round#6

T7   A caretaker is giving candy to his two babies. Every minute, he gives a candy to one of his two babies at random. The five possible moods for the babies to be in, from saddest to happiest, are "upset," "sad," "okay," "happy," and "delighted." A baby gets happier by one mood when they get a candy and gets sadder by one mood when the other baby gets one. Both babies start at the "okay" state, and a baby will start crying if they don't get a candy when they're already "upset". The probability that 10 minutes pass without either baby crying can be expressed as $\frac{p}{q}$. Compute $p+q$.

2022 CCA Math Bonanza Team Round#7

T8   Let n be a set of integers. $S(n)$ is defined as the sum of the elements of n. $T = \{1,2,3,4,5,6,7,8,9\}$ and A and B are subsets of T such that A $\cup B = T$ and A $\cap B = \varnothing$. The probability that $S(A) \geq 4S(B)$ can be expressed as $\frac{p}{q}$. Compute $p+q$.

2022 CCA Math Bonanza Team Round#8

T9   Equilateral octagon $A_1A_2A_3A_4A_5A_6A_7A_8$ is constructed such that $A_1A_3A_5A_7$ is a square of side length $\sqrt{2}$ and $A_2A_4A_6A_8$ is a square of side length 4/3. For each vertex A_i of the octagon, let B_i be the intersection of lines $A_{i+1}A_{i+2}$ and $A_{i-1}A_{i-2}$, where $A_{i-8} = A_i = A_{i+8}$. Compute $[B_1B_2B_3B_4B_5B_6B_7B_8]^2$.

2022 CCA Math Bonanza Team Round#9

T10 Evan, Larry, and Alex are drawing whales on the whiteboard. Evan draws 10 whales, Larry draws 15 whales, and Alex draws 20 whales. Michelle then starts randomly erasing whales one by one. The probability that she finishes erasing Larry's whales first can be expressed as $\frac{p}{q}$. Compute $p + q$.

2022 CCA Math Bonanza Team Round#10

– Lightning Round

L1.1 Given

$$a = bc$$
$$b = ca$$
$$c = a + b$$
$$c > a$$

Evaluate $a + b + c$.

2022 CCA Math Bonanza Lightning Round 1.1

L1.2 Xonathan Jue goes to the casino with exactly $1000. Each week, he has a $1/3$ chance of breaking even and $2/3$ chance of losing $500. Evaluate the expected amount of weeks before he loses everything.

2022 CCA Math Bonanza Lightning Round 1.2

L1.3 The area of the region bound by the equations $y = 2\cos(4x)$, $y = \sin(2x) + 10$, $x = 0$, and $x = 5\pi$ can be expressed as $x\pi$ for some value of x. Evaluate x.

2022 CCA Math Bonanza Lightning Round 1.3

L1.4 Jongol and Gongol are writing calculus questions and grading tests. They want to write 90 calculus problems and they have 120 tests to grade. Jongol can write 3 questions per minute or grade 4 tests per minute. Gongol can write 1 question per minute or grade 2 tests per minute. Evaluate the shortest possible time, in minutes, for them to complete the two tasks.

2022 CCA Math Bonanza Lightning Round 1.4

L2.1 Given that a duck found that $5 - 2\sqrt{3}i$ is one of the roots of $-259 + 107x - 17x^2 + x^3$, what is the sum of the real parts of the other two roots?

2022 CCA Math Bonanza Lightning Round 2.1

L2.2   A rectangle $ABCD$ has side lengths $AB = 6$ miles and $BC = 9$ miles. A pigeon hovers at point P, which is 5 miles above some randomly chosen point inside $ABCD$. Given that the expected value of

$$AP^2 + CP^2 - BP^2 - DP^2$$

can be expressed as $\frac{a}{b}$, what is ab?

2022 CCA Math Bonanza Lightning Round 2.2

L2.3   Given that the height of a greater sage grouse flying through the air is defined by the function $64x - x^2$ for $0 < x < 64$, what is the first time at which the bird reaches a height of 903?

2022 CCA Math Bonanza Lightning Round 2.3

L2.4   10 geese are numbered 1-10. One goose leaves the pack, and the remaining nine geese assemble in a symmetric V-shaped formation with four geese on each side. Given that the product of the geese on both halves of the "V" are the same, what is the sum of the possible values of the goose that left?

2022 CCA Math Bonanza Lightning Round 2.4

L3.1   Kongol rolls two fair 6-sided die. The probability that one roll is a divisor of the other can be expressed as $\frac{p}{q}$. Determine $p + q$.

2022 CCA Math Bonanza Lightning Round 3.1

L3.2   In the following diagram, $AB = 1$. The radius of the circle with center C can be expressed as $\frac{p}{q}$. Determine $p + q$.

2022 CCA Math Bonanza Lightning Round 3.2

L3.3   Determine the sum of all positive integers $n < 100$ satisfying the following expression.

$$\sum_{k=0}^{\lfloor \log_{10} n \rfloor} \frac{1}{10^k} \left(n \,(\mathrm{mod}\; 10^{k+1}) - n \,(\mathrm{mod}\; 10^k) \right) = \prod_{k=0}^{\lfloor \log_{10} n \rfloor} \frac{1}{10^k} \left(n \,(\mathrm{mod}\; 10^{k+1}) - n \,(\mathrm{mod}\; 10^k) \right).$$

Here, $\sum$ and $\prod$ represent sum and product, respectively.

2022 CCA Math Bonanza Lightning Round 3.3

L3.4   A merganser mates every 7th day, a scaup mates every 11th day, and a gadwall mates every 13th day. A merganser, scaup, and gadwall all mate on Day 0. On Days N, N+1, and N+2 the merganser, scaup, and gadwall mate in some order with no two birds mating on the same day. Determine the smallest possible value of N.

2022 CCA Math Bonanza Lightning Round 3.4

L4.1   There are 99 seagulls labeled 2-100 and 100 bagels labeled 1-100. Starting from Seagull 2, each Seagull N eats $\frac{1}{N}$ of whatever remains of each Bagel I where N divides I. How many bagels still have more than $\frac{2}{3}$ of their original size after Seagull 100 finishes eating?

2022 CCA Math Bonanza Lightning Round 4.1

L4.2   A rhombicosidodecahedron is an Archimedean solid, one of thirteen convex isogonal nonprismatic solids constructed of two or more types of regular polygon faces. It has 20 regular triangular faces, 30 square faces, and 12 regular pentagonal faces, as shown below. How many rotational symmetries does a rhombicosidodecahedron have?

2022 CCA Math Bonanza Lightning Round 4.2

L4.3   Ethan Song and Bryan Guo are playing an unfair game of rock-paper-scissors. In any game, Ethan has a 2/5 chance to win, 2/5 chance to tie, and 1/5 chance to lose. How many games is Ethan expected to win before losing?

2022 CCA Math Bonanza Lightning Round 4.3

L4.4   Lukas Nepomuceno draws 5 congruent circles equally spaced around a 6th, and colors all of them 1 of 3 colors. Assume that rotations and reflections of colorings are indistinguishable. How many distinct colorings are there?

2022 CCA Math Bonanza Lightning Round 4.4

L5.1   Alistar wants to wreak havoc on Jhin's yard, which is a 2D plane of grass. First, he selects a number n, randomly and uniformly from $[0, 1]$, and then he eats all grass within n meters from where he's standing. He then moves 2 meters in a random direction, and repeats his process. He stops if any of the grass that he wants to eat (or, in other words, in his intended eating territory) is already eaten. Estimate the amount of grass Alistar is expected to eat. An estimate E earns $\frac{2}{1+|A-E|}$ points, where A is the actual answer.

2022 CCA Math Bonanza Lightning Round 5.1

L5.2   Estimate the range of the submissions for this problem. Your answer must be between $[0, 1000]$. An estimate E earns $\frac{2}{1+0.05|A-E|}$ points, where A is the actual answer.

2022 CCA Math Bonanza Lightning Round 5.2

L5.3   Estimate the number of times a one-digit answer (0, 1, 2, 3, 4, 5, 6, 7, 8, or 9) has been submitted as an answer for any question by any team in the first 4 sets of this competition's lightning round. An estimate E earns $\frac{2}{1+|log_2(A)-log_2(E)|}$ points, where A is the actual answer.

2022 CCA Math Bonanza Lightning Round 5.3

L5.4 Five points are selected within a unit circle at random. Estimate the minimum distance between any pair of points. An estimate E earns $\frac{2}{1+|log_2(A)-log_2(E)|}$ points, where A is the actual answer.

2022 CCA Math Bonanza Lightning Round 5.4

– Tiebreaker Round

TB1 How many positive integer factors does the following expression have?

$$\sum_{n=1}^{999} \log_{10}\left(\frac{n+1}{n}\right)$$

2022 CCA Math Bonanza Tiebreaker Round#1

TB2 Determine the last three digits of 374^{2022}.

2022 CCA Math Bonanza Tiebreaker Round#2

TB3 Given that $(2\cos^2 7.5 - \cos 75 - 1)^2$ can be expressed as $\frac{p}{q}$, what is $p+q$?

2022 CCA Math Bonanza Tiebreaker Round#3

TB4 Let $f(x)$ be a function such that $f(1) = 1234$, $f(2) = 1800$, and $f(x) = f(x-1) + 2f(x-2) - 1$ for all integers x. Evaluate the number of divisors of

$$\sum_{i=1}^{2022} f(i)$$

2022 CCA Math Bonanza Tiebreaker Round#4

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