

CMO- Caucasus Mathematical Olympiad (Note: in this year, CMO and EGMO TST were the same tests)

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G1 Let ABC be an isosceles triangle with $AC = BC$ and circumcircle k . The point D lies on the shorter arc of k over the chord BC and is different from B and C . Let E denote the intersection of CD and AB . Prove that the line through B and C is a tangent of the circumcircle of the triangle BDE .

(Karl Czakler)

A2 Let a, b and c be pairwise different natural numbers. Prove $\frac{a^3+b^3+c^3}{3} \geq abc + a + b + c$.
When does equality holds?

(Karl Czakler)

C3 Suppose $n \geq 3$ is an integer. There are n grids on a circle. We put a stone in each grid. Find all positive integer n , such that we can perform the following operation $n - 2$ times, and then there exists a grid with $n - 1$ stones in it:

- Pick a grid A with at least one stone in it. And pick a positive integer $k \leq n - 1$. Take all stones in the k -th grid after A in anticlockwise direction. And put then in the k -th grid after A in clockwise direction.

N4 Let $n \geq 1$ be a positive integer. We say that an integer k is a *fan* of n if $0 \leq k \leq n - 1$ and there exist integers $x, y, z \in \mathbb{Z}$ such that

$$\begin{aligned}x^2 + y^2 + z^2 &\equiv 0 \pmod{n}; \\xyz &\equiv k \pmod{n}.\end{aligned}$$

Let $f(n)$ be the number of fans of n . Determine $f(2020)$.