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G1 Let ABC be a triangle with $AB < AC < BC$. On the side BC we consider points D and E such that $BA = BD$ and $CE = CA$. Let K be the circumcenter of triangle ADE and let F, G be the points of intersection of the lines AD, KC and AE, KB respectively. Let ω_1 be the circumcircle of triangle KDE, ω_2 the circle with center F and radius FE, and ω_3 the circle with center G and radius GD.
Prove that ω_1, ω_2, and ω_3 pass through the same point and that this point of intersection lies on the line AK.

A2 Find all functions $f : R \to R$ with $f(x + yf(x + y)) = y^2 + f(x)f(y)$ for all $x, y \in R$.

C3 In an exotic country, the National Bank issues coins that can take any value in the interval $[0, 1]$. Find the smallest constant $c > 0$ such that the following holds, no matter the situation in that country:

[i]Any citizen of the exotic country that has a finite number of coins, with a total value of no more than 1000, can split those coins into 100 boxes, such that the total value inside each box is at most c.[/i]

N4* A natural number n is given. Determine all $(n-1)$-tuples of nonnegative integers $a_1, a_2, ..., a_{n-1}$ such that

$$\lfloor\frac{m}{2^n-1}\rfloor + \lfloor\frac{2m+a_1}{2^n-1}\rfloor + \lfloor\frac{2^2m+a_2}{2^n-1}\rfloor + \lfloor\frac{2^3m+a_3}{2^n-1}\rfloor + ... + \lfloor\frac{2^{n-1}m+a_{n-1}}{2^n-1}\rfloor = m$$

holds for all $m \in \mathbb{Z}$.
