Turkey EGMO TST 2015

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Day 1 February 12th

1 a is a real number. Find the all (x,y) real number pairs satisfy;

$$y^2 = x^3 + (a-1)x^2 + a^2x$$

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2 Let D be the midpoint of the side BC of a triangle ABC and P be a point inside the ABD satisfying $\angle PAD = 90^\circ - \angle PBD = \angle CAD$. Prove that $\angle PQB = \angle BAC$, where Q is the intersection point of the lines PC and AD.

3 Given a 2015-tuple $(a_1, a_2, \ldots, a_{2015})$ in each step we choose two indices $1 \le k, l \le 2015$ with a_k even and transform the 2015-tuple into $(a_1, \ldots, \dfrac{a_k}{2}, \ldots, a_l + \dfrac{a_k}{2}, \ldots, a_{2015})$. Prove that starting from $(1, 2, \ldots, 2015)$ in finite number of steps one can reach any permutation of $(1, 2, \ldots, 2015)$.

Day 2 February 13th

4 Find the all (m,n) integer pairs satisfying $m^4 + 2n^3 + 1 = mn^3 + n$.

5 Let $a \ge b \ge 0$ be real numbers. Find the area of the region defined as;

$K = \{(x,y) : x \ge y \ge 0$ and $\forall n$ positive integers satisfy $a^n + b^n \ge x^n + y^n\}$

in the cordinate plane.

6 In a party attended by 2015 guests among any 5 guests at most 6 handshakes had been exchanged. Determine the maximal possible total number of handshakes.