National Math Olympiad (Second Round) 2022

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– Day 1

1 Let E and F on AC and AB respectively in $\triangle ABC$ such that $DE||BC$ then draw line l through A such that $l||BC$ let D' and E' reflection of D and E to l respectively prove that $D'B, E'C$ and l are congruence.

2 Find all functions $f:\mathbb{R}\rightarrow\mathbb{R}$ such that for any real value of x, y we have:

$$f(xf(y)+f(x)+y)=xy+f(x)+f(y)$$

3 Take a $n \times n$ chess page.Determine the n such that we can put the numbers $1,2,3,\ldots,n$ in the squares of the page such that we know the following two conditions are true:

a) for each row we know all the numbers $1,2,3,\ldots,n$ have appeared on it and the numbers that are in the black squares of that row have the same sum as the sum of the numbers in the white squares of that row.

b) for each column we know all the numbers $1,2,3,\ldots,n$ have appeared on it and the numbers that are in the black squares in that column have the same sum as the sum of the numbers in the white squares of that column.

– Day 2

4 There is an $n*n$ table with some unit cells colored black and the others are white.
In each step , Amin takes a row with exactly one black cell in it , and color all cells in that black cell's $column$ red.
While Ali , takes a $column$ with exactly one black cell in it , and color all cells in that black cell's row red.
Prove that Amin can color all the cells red , iff Ali can do so.

5 define $(a_n)_{n\in\mathbb{N}}$ such that $a_1=2$ and

$$a_{n+1}=\left(1+\frac{1}{n}\right)^n\times a_n$$

Prove that there exists infinite number of n such that $\frac{a_1a_2\ldots a_n}{n+1}$ is a square of an integer.

6 we have an isogonal triangle ABC such that $BC = AB$. take a random P on the altitude from B to AC.
The circle (ABP) intersects AC second time in M. Take N such that it's on the segment AC and $AM = NC$ and $M \neq N$.The second intersection of NP and circle (APB) is X , ($X \neq P$) and the second intersection of AB and circle (APN) is Y ,($Y \neq A$).The tangent from A to the circle (APN) intersects the altitude from B at Z.
Prove that CZ is tangent to circle (PXY).