APMO 2022

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1 Find all pairs (a, b) of positive integers such that a^3 is multiple of b^2 and $b-1$ is multiple of $a-1$.

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2 Let ABC be a right triangle with $\angle B = 90^\circ$. Point D lies on the line CB such that B is between D and C. Let E be the midpoint of AD and let F be the seconf intersection point of the circumcircle of $\triangle ACD$ and the circumcircle of $\triangle BDE$. Prove that as D varies, the line EF passes through a fixed point.

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3 Find all positive integers $k < 202$ for which there exist a positive integers n such that

$$\left\{\frac{n}{202}\right\} + \left\{\frac{2n}{202}\right\} + \cdots + \left\{\frac{kn}{202}\right\} = \frac{k}{2}$$

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4 Let n and k be positive integers. Cathy is playing the following game. There are n marbles and k boxes, with the marbles labelled 1 to n. Initially, all marbles are placed inside one box. Each turn, Cathy chooses a box and then moves the marbles with the smallest label, say i, to either any empty box or the box containing marble $i+1$. Cathy wins if at any point there is a box containing only marble n.
Determine all pairs of integers (n, k) such that Cathy can win this game.

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5 Let a, b, c, d be real numbers such that $a^2 + b^2 + c^2 + d^2 = 1$. Determine the minimum value of $(a-b)(b-c)(c-d)(d-a)$ and determine all values of (a, b, c, d) such that the minimum value is achived.
