Tests consisting of IMOSL problems are not shown.

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Test 2 January 17, 2022

1 A triangle ABC with orthocenter H is given. P is a variable point on line BC. The perpendicular to BC through P meets BH, CH at X, Y respectively. The line through H parallel to BC meets AP at Q. Lines QX and QY meet BC at U, V respectively. Find the shape of the locus of the incenters of the triangles QUV.

2 The numbers a, b, and c are real. Prove that

$$(a^5+b^5+c^5+a^3c^2+b^3a^2+c^3b^2)^2 \geq 4(a^2+b^2+c^2)(a^5b^3+b^5c^3+c^5a^3)$$

3 A class has 30 students. To celebrate 'Tu BiShvat' each student chose some dried fruits out of n different kinds. Say two students are friends if they both chose from the same type of fruit. Find the minimal n so that it is possible that each student has exactly 6 friends.

Test 7 May 17, 2022

1 Let $n>1$ be an integer. Find all $r\in\mathbb{R}$ so that the system of equations in real variables $x_1, x_2, \dots, x_n$:

$$\begin{aligned}&(r\cdot x_1-x_2)(r\cdot x_1-x_3)\dots(r\cdot x_1-x_n)=\\ =&(r\cdot x_2-x_1)(r\cdot x_2-x_3)\dots(r\cdot x_2-x_n)=\\ &\vdots\\ =&(r\cdot x_n-x_1)(r\cdot x_n-x_2)\dots(r\cdot x_n-x_{n-1})\end{aligned}$$

has a solution where the numbers $x_1, x_2, \dots, x_n$ are pairwise distinct.

2 Define a **ring** in the plane to be the set of points at a distance of at least r and at most R from a specific point O, where $r<R$ are positive real numbers. Rings are determined by the three parameters (O, R, r). The area of a ring is labeled S. A point in the plane for which both its coordinates are integers is called an integer point.

a) For each positive integer n, show that there exists a ring not containing any integer point, for which $S>3n$ and $R<2^{2^n}$.

b) Show that each ring satisfying $100\cdot R<S^2$ contains an integer point.

3 In triangle ABC, the angle bisectors are BE and CF (where E, F are on the sides of the triangle), and their intersection point is I. Point N lies on the circumcircle of AEF, and the angle $\angle IAN$ is right. The circumcircle of AEF meets the line NI a second time at the point L. Show that the circumcenter of AIL lies on line BC.

Test 9 June 29, 2022

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1 Bilbo, Gandalf, and Nitzan play the following game. First, Nitzan picks a whole number between 1 and 2^{2022} inclusive and reveals it to Bilbo. Bilbo now compiles a string of length 4044 built from the three letters a, b, c. Nitzan looks at the string, chooses one of the three letters a, b, c, and removes from the string all instances of the chosen letter. Only then is the string revealed to Gandalf. He must now guess the number Nitzan chose.

Can Bilbo and Gandalf work together and come up with a strategy beforehand that will always allow Gandalf to guess Nitzan's number correctly, no matter how he acts?

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2 Let $f : \mathbb{Z}^2 \to \mathbb{R}$ be a function.
It is known that for any integer C the four functions of x

$$f(x, C), f(C, x), f(x, x + C), f(x, C - x)$$

are polynomials of degree at most 100. Prove that f is equal to a polynomial in two variables and find its maximal possible degree.

[i]Remark: The degree of a bivariate polynomial $P(x, y)$ is defined as the maximal value of $i + j$ over all monomials $x^i y^j$ appearing in P with a non-zero coefficient.[/i]

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3 Scalene triangle ABC has incenter I and circumcircle Ω with center O. H is the orthocenter of triangle BIC, and T is a point on Ω for which $\angle ATI = 90^\circ$. Circle (AIO) intersects line IH again at X. Show that the lines AX, HT intersect on Ω.
