**VII Iberoamerican Interuniversitary Mathematics Competition - Mexico**
www.artofproblemsolving.com/community/c305025
by Ozc

**Problem 1** Find the real number $a$ such that the integral

$$\int_a^{a+8} e^{-x} e^{-x^2} dx$$

attain its maximum.

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**Problem 2** Find all polynomials $P(x)$ with real coefficients that satisfy the identity

$$P(x^3 - 2) = P(x)^3 - 2,$$

for every real number $x$.

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**Problem 3** Consider the matrices

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}.$$

Let $k \geq 1$ an integer. Prove that for any nonzero $i_1, i_2, \dots, i_{k-1}, j_1, j_2, \dots, j_k$ and any integers $i_0, i_k$ it holds that

$$A^{i_0} B^{j_1} A^{i_1} B^{j_2} \cdots A^{i_{k-1}} B^{i_k} A^{i_k} \neq I.$$

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**Problem 4** Let $f : \mathbb{R} \to \mathbb{R}$ a continuos function and $\alpha$ a real number such that

$$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = \alpha.$$

Prove that for any $r > 0$, there exists $x, y \in \mathbb{R}$ such that $y - x = r$ and $f(x) = f(y)$.

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**Problem 5** There are $n$ people seated on a circular table that have seats numerated from 1 to $n$ clockwise. Let $k$ be a fix integer with $2 \leq k \leq n$. The people can change their seats. There are two types of moves permitted:

1. Each person moves to the next seat clockwise.
2. Only the ones in seats 1 and $k$ exchange their seats.

Determine, in function of $n$ and $k$, the number of possible configurations of people in the table that can be attain by using a sequence of permitted moves.

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**Problem 6** Show that there exists a real $C > 1$ that satisfy the following property: if $n > 1$ and $a_0 < a_1 < \cdots < a_n$ are positive integers such that $\frac{1}{a_0}, \frac{1}{a_1}, \dots, \frac{1}{a_n}$ are in arithmetic progression, then $a_0 > C^n$.

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