ELMO Problems 2022

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–	Day 1
1	Let $n > 1$ be an integer. The numbers $1, 2, \ldots, n$ are written on a board. Aliceurill and Bobasaur take turns circling an uncircled number on the board, with Aliceurill going first. When the product of the circled numbers becomes a multiple of n, the game ends and the last player to have circled a number loses. For which values of n can Bobasaur guarantee victory? *Max Lu*
2	Find all monic nonconstant polynomials P with integer coefficients for which there exist positive integers a and m such that for all positive integers $n \equiv a \pmod{m}$, $P(n)$ is nonzero and $$2022 \cdot \frac{(n+1)^{n+1} - n^n}{P(n)}$$ is an integer. *Jaedon Whyte, Luke Robitaille, and Pitchayut Saengrungkongka*
3	Let $\mathcal{P}$ be a regular 2022-gon with area 1. Find a real number c such that, if points A and B are chosen independently and uniformly at random on the perimeter of $\mathcal{P}$, then the probability that $AB \geq c$ is $\frac{1}{2}$. *Espen Slettnes*
–	Day 2
4	Let $ABCDE$ be a convex pentagon such that $\triangle ABE$, $\triangle BEC$, and $\triangle EDB$ are similar (with vertices in order). Lines BE and CD intersect at point T. Prove that line AT is tangent to the circumcircle of $\triangle ACD$. *Holden Mui*
5	Let $n \geq 3$ be a positive integer. There are n^3 users on a social media network called *Everyone Likes Meeting Online* (ELMO), and some pairs of these users are *buddies*. A set of at least three ELMO users forms an *ELMOClub* if and only if all pairs of members of the set are buddies. It is known that among every n users, some three form an ELMOclub. Prove that there is an ELMOclub with five members. *Luke Robitaille*

6 Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that, for all integers m and n,

$$f(f(m) - n) + f(f(n) - m) = f(m + n).$$

Espen Slettnes and Luke Robitaille