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- 1 Find all pairs of positive integers (a, b) such that

$$11ab \leq a^3 - b^3 \leq 12ab.$$

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- 2 Let ABC be an acute triangle such that $AH = HD$, where H is the orthocenter of ABC and $D \in BC$ is the foot of the altitude from the vertex A . Let ℓ denote the line through H which is tangent to the circumcircle of the triangle BHC . Let S and T be the intersection points of ℓ with AB and AC , respectively. Denote the midpoints of BH and CH by M and N , respectively. Prove that the lines SM and TN are parallel.

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- 3 Find all quadruples of positive integers (p, q, a, b) , where p and q are prime numbers and $a > 1$, such that

$$p^a = 1 + 5q^b.$$

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- 4 We call an even positive integer n nice if the set $\{1, 2, \dots, n\}$ can be partitioned into $\frac{n}{2}$ two-element subsets, such that the sum of the elements in each subset is a power of 3. For example, 6 is nice, because the set $\{1, 2, 3, 4, 5, 6\}$ can be partitioned into subsets $\{1, 2\}$, $\{3, 6\}$, $\{4, 5\}$. Find the number of nice positive integers which are smaller than 3^{2022} .
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