**Joint Kosovo & Albania Mathematical Olympiad for children in grades 7-9**

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by bsf714

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| **–** | Grades 7-8 |
| **1** | Find all pairs of integers $(m, n)$ such that $$m+n=3(mn+10).$$ |
| **2** | Consider a $5\times 5$ grid with $25$ cells. What is the least number of cells that should be colored, such that every $2\times 3$ or $3\times 2$ rectangle in the grid has at least two colored cells? |
| **3** | Let $ABCD$ be a square and let $M$ be the midpoint of $BC$. Let $X$ and $Y$ be points on the segments $AB$ and $CD$, respectively. Prove that $\angle XMY = 90^\circ$ if and only if $BX+CY=XY$. *Note: In the competition, students were only asked to prove the 'only if' direction.* |
| **4** | Let $A$ be the set of natural numbers $n$ such that the distance of the real number $n\sqrt{2022}-\frac{1}{3}$ from the nearest integer is at most $\frac{1}{2022}$. Show that the equation $$20x+21y=22z$$ has no solutions over the set $A$. |
| **–** | Grade 9 |
| **–** | Let $a>0$. If the inequality $22<ax<222$ holds for precisely $10$ positive integers $x$, find how many positive integers satisfy the inequality $222<ax<2022$? *Note: The first 8 problems of the competition are questions which the contestants are expected to solve quickly and only write the answer of. This problem turned out to be a lot more difficult than anticipated for an answer-only question.* |
| **1** | If $(2^x-4^x)+(2^{-x}-4^{-x})=3$, find the numerical value of the expression $$(8^x+3\cdot 2^x)+(8^{-x}+3\cdot 2^{-x}).$$ |
| **2** | Let $ABC$ be an acute triangle. Let $D$ be a point on the line parallel to $AC$ that passes through $B$, such that $\angle BDC = 2\angle BAC$ as well as such that $ABDC$ is a convex quadrilateral. Show that $BD+DC=AC$. |

**3** Is it possible to partition $\{1, 2, 3, \ldots, 28\}$ into two sets $A$ and $B$ such that both of the following conditions hold simultaneously:

(i) the number of odd integers in $A$ is equal to the number of odd integers in $B$;

(ii) the difference between the sum of squares of the integers in $A$ and the sum of squares of the integers in $B$ is $16$?

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**4** Consider $n > 9$ lines on the plane such that no two lines are parallel. Show that there exist at least $n/9$ lines such that the angle between any two of the lines is at most $20°$.

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