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N1 Find all positive integers a, b, c such that $ab+1$, $bc+1$, and $ca+1$ are all equal to factorials of some positive integers.

Proposed by *Nikola Velov, Macedonia*

A2 For positive real numbers a, b, c, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq \frac{3}{abc}$ is true. Prove that:

$$\frac{a^2+b^2}{a^2+b^2+1}+\frac{b^2+c^2}{b^2+c^2+1}+\frac{c^2+a^2}{c^2+a^2+1} \geq 2$$

G3 In acute, scalene Triangle ABC, H is orthocenter,BD and CE are heights. X, Y are reflection of A from D,E respectively such that the pointsX, Y are on segments DC and EB. The intersection of circles HXY and ADE is F. ($F \neq H$). Prove thatAF intersects middle point of BC. (M in the diagram is Midpoint of BC)

C4 n is a natural number. Given $3n \cdot 3n$ table, the unit cells are colored white and black such that starting from the left up corner diagonals are colored in pure white or black in ratio of 2:1 respectively. (See the picture below). In one step any chosen $2 \cdot 2$ square's white cells are colored orange, orange are colored black and black are colored white. Find all n such that with finite steps, all the white cells in the table turns to black, and all black cells in the table turns to white. (From starting point)

C5? Alice and Bob play a game together as a team on a 100×100 board with all unit squares initially white. Alice sets up the game by coloring exactly k of the unit squares red at the beginning. After that, a legal move for Bob is to choose a row or column with at least 10 red squares and color all of the remaining squares in it red. What is the
smallest k such that Alice can set up a game in such a way that Bob can color the entire board red after finitely many moves?

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