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–	Grade 9
1	Ana has 22 coins. She can take from her friends either 6 coins or 18 coins, or she can give 12 coins to her friends. She can do these operations many times she wants. Find the least number of coins Ana can have.
2	Show that for any positive real numbers a and b the following inequality hold, $$\frac{a(a+1)}{b+1}+\frac{b(b+1)}{a+1}\geq a+b.$$
3	Let $ABCD$ be a parallelogram and l the line parallel to AC which passes through D. Let E and F points on l such that $DE=DF=DB$. Show that EA, FC and BD are concurrent.
4	Find all prime numbers p and q such that $pq-p-q+3$ is a perfect square.
–	Grade 10
1	Ana has a scale that shows which side weight more or if both side are equal. She has 4 weights which look the same but they weight $1001g, 1002g, 1004g$ and $1005g$, respectively. Is it possible for Ana to find out the weight of each of them with only 4 measurements?
2	Let ABC be an isosceles triangle with $CA=CB$ and $\angle ACB=20^\circ$. Let D be a point on side CA such that $\angle ADB=30^\circ$. Show that $AB=CD$.
3	Let a, b and c be positive integers such that $a!+b+c, b!+c+a$ and $c!+a+b$ are prime numbers. Show that $\frac{a+b+c+1}{2}$ is also a prime number.
4	Let a, b and c be positive real numbers such that $a+b+c+3abc\geq (ab)^2+(bc)^2+(ca)^2+3$. Show that the following inequality hold, $$\frac{a^3+b^3+c^3}{3}\geq\frac{abc+2021}{2022}.$$
–	Grade 11

1	22 light bulbs are given. Each light bulb is connected to exactly one switch, but a switch can be connected to one or more light bulbs. Find the least number of switches we should have such that we can turn on whatever number of light bulbs.
2	Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that for all real numbers x and y, $$f(f(x-y) - yf(x)) = xf(y).$$
3	Find all positive integers n such that $10^n + 3^n + 2$ is a palindrom number.
4	Assume that in the $\triangle ABC$ there exists a point D on BC and a line l passing through A such that l is tangent to (ADC) and l bisects BD. Prove that $a\sqrt{2} \geq b + c$.
–	Grade 12
1	Find all real numbers a, b and c such that $a + bc = b + ca = c + ab$.
2	Let be given n positive integer. Lets write with a_n the number of positive integer pairs (x, y) such that $x + y$ is even and $1 \leq x \leq y \leq n$. Lets write with b_n the number of positive integer pairs (x, y) such that $x + y \leq n + 1$ and $1 \leq x \leq y \leq n$.
3	Let $\triangle ABC$ be a triangle and D be a point in line BC such that AD bisects $\angle BAC$. Furthermore, let F and G be points on the circumcircle of $\triangle ABC$ and $E \neq D$ point in line BC such that $AF = AE = AD = AG$. If X and Y are the feet of perpendiculars from D to EF and EG, respectively. Prove that $XY \parallel AD$.
4	Find all positive integers k, m and n such that $k! + 3^m = 3^n$