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by dangerousliri, Lukaluce

1 Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that for all real numbers x and y,

$$f(x^2) + 2f(xy) = xf(x+y) + yf(x).$$

Proposed by Dorlir Ahmeti, Kosovo

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2 Find all positive integers a, b, c such that $ab+1$, $bc+1$, and $ca+1$ are all equal to factorials of some positive integers.

Proposed by *Nikola Velov, Macedonia*

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3 Let ABC be a triangle and D point on side BC such that AD is angle bisector of angle $\angle BAC$. Let E be the intersection of the side AB with circle ω_1 which has diameter CD and let F be the intersection of the side AC with circle ω_2 which has diameter BD. Suppose that there exist points $P \in \omega_1$ and $Q \in \omega_2$ such that E, P, Q and F are collinear and on this order. Prove that AD, BQ and CP are concurrent.

Proposed by Dorlir Ahmeti, Kosovo and Noah Walsh, U.S.A.

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4 On a board, Ana writes a different integers, while Ben writes b different integers. Then, Ana adds each of her numbers with with each of Ben's numbers and she obtains c different integers. On the other hand, Ben substracts each of his numbers from each of Ana's numbers and he gets d different integers.

For each integer n , let $f(n)$ be the number of ways that n may be written as sum of one number of Ana and one number of Ben.

a) Show that there exist an integer n such that,

$$f(n) \geq \frac{ab}{c}.$$

b) Does there exist an integer n such that,

$$f(n) \geq \frac{ab}{d}?$$

Proposed by Besfort Shala, Kosovo
