Spain Mathematical Olympiad 2016

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by gavrilos, ESCHen99, Sumgato

–	Day 1
1	Two real number sequences are guiven, one arithmetic $(a_n)_{n\in\mathbb{N}}$ and another geometric sequence $(g_n)_{n\in\mathbb{N}}$ none of them constant. Those sequences verifies $a_1 = g_1 \neq 0$, $a_2 = g_2$ and $a_{10} = g_3$. Find with proof that, for every positive integer p, there is a positive integer m, such that $g_p = a_m$.
2	Given a positive prime number p. Prove that there exist a positive integer α such that $p\|\alpha(\alpha-1)+3$, if and only if there exist a positive integer β such that $p\|\beta(\beta-1)+25$.
3	In the circumscircle of a triangle ABC, let A_1 be the point diametrically opposed to the vertex A. Let A' the intersection point of AA' and BC. The perpendicular to the line AA' from A' meets the sides AB and AC at M and N, respectively. Prove that the points A, M, A_1 and N lie on a circle which has the center on the height from A of the triangle ABC.
–	Day 2
4	Let m be a positive integer and a and b be distinct positive integers strictly greater than m^2 and strictly less than m^2+m. Find all integers d such that $m^2 < d < m^2+m$ and d divides ab.
5	From all possible permutations from $(a_1, a_2, ..., a_n)$ from the set $\{1, 2, .., n\}$, $n \geq 1$, consider the sets that satisfies the $2(a_1 + a_2 + ... + a_m)$ is divisible by m, for every $m = 1, 2, ..., n$. Find the total number of permutations.
6	Let $n \geq 2$ an integer. Find the least value of γ such that for any positive real numbers $x_1, x_2, ..., x_n$ with $x_1 + x_2 + ... + x_n = 1$ and any real $y_1 + y_2 + ... + y_n = 1$ and $0 \leq y_1, y_2, ..., y_n \leq \frac{1}{2}$ the following inequality holds: $$x_1x_2...x_n \leq \gamma\left(x_1y_1 + x_2y_2 + ... + x_ny_n\right)$$