Cono Sur Olympiad 2022

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–	Day 1
1	A positive integer is *happy* if: 1. All its digits are different and not 0, 2. One of its digits is equal to the sum of the other digits. For example, 253 is a *happy* number. How many *happy* numbers are there?
2	Given is a triangle ABC with incircle ω, tangent to BC, CA, AB at D, E, F. The perpendicular from B to BC meets EF at M, and the perpendicular from C to BC meets EF at N. Let DM and DN meet ω at P and Q. Prove that $DP = DQ$.
3	Prove that for every positive integer n there exists a positive integer k, such that each of the numbers $k, k^2, \ldots, k^n$ have at least one block of 2022 in their decimal representation. For example, the numbers 4**2022**13 and 544**2022**1**2022** have at least one block of 2022 in their decimal representation.
–	Day 2
4	Ana and Beto play on a grid of 2022×2022. Ana colors the sides of some squares on the board red, so that no square has two red sides that share a vertex. Next, Bob must color a blue path that connects two of the four corners of the board, following the sides of the squares and not using any red segments. If Beto succeeds, he is the winner, otherwise Ana wins. Who has a winning strategy?
5	An integer $n > 1$, whose positive divisors are $1 = d_1 < d_2 < \cdots < d_k = n$, is called *southern* if all the numbers $d_2 - d_1, d_3 - d_2, \cdots, d_k - d_{k-1}$ are divisors of n. a) Find a positive integer that is *not southern* and has exactly 2022 positive divisors that are *southern*. b) Show that there are infinitely many positive integers that are *not southern* and have exactly 2022 positive divisors that are *southern*.
6	On a blackboard the numbers $1, 2, 3, \ldots, 170$ are written. You want to color each of these numbers with k colors $C_1, C_2, \ldots, C_k$, such that the following condition is satisfied: for each i with $1 \leq i < k$, the sum of all numbers with color C_i divide the sum of all numbers with color C_{i+1}. Determine the largest possible value of k for which it is possible to do that coloring.