

Czech And Slovak Mathematical Olympiad, Round III, Category A, 1956

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by byk7

- 1 Find all $x, y \in (0, \frac{\pi}{2})$ such that

$$\frac{\cos x}{\cos y} = 2 \cos^2 y,$$
$$\frac{\sin x}{\sin y} = 2 \sin^2 y.$$

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- 2 In a given plane ρ consider a convex quadrilateral $ABCD$ and denote $E = AC \cap BD$. Moreover, consider a point $V \notin \rho$. On rays VA, VB, VC, VD find points A', B', C', D' respectively such that E, A', B', C', D' are coplanar and $A'B'C'D'$ is a parallelogram. Discuss conditions of solvability.

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- 3 Find all real pairs x, y such that

$$x - |y + 1| = 1,$$
$$x^2 + y = 10.$$

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- 4 Let a semicircle AB be given and let X be an inner point of the arc. Consider a point Y on ray XA such that $XY = XB$. Find the locus of all points Y when X moves on the arc AB (excluding the endpoints).
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