geometry Rounds from Stanford and Rice Math Tournament, many years they shared problems (R= Rice, for different problems)

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by parmenides51, tc1729, Mrdavid445

2001.1 Find the coordinates of the points of intersection of the graphs of the equations $y = |2x| - 2$ and $y = -|2x| + 2$.

2001.2 Jacques is building an igloo for his dog. The igloo's inside and outside are both perfectly hemispherical. The interior height at the center is 2 feet. The igloo has no door yet and contains $\frac{254}{2187}\pi$ cubic yards of hand-packed snow. What is the circumference of the igloo at its base in feet?

2001.3 Find the area of the convex quadrilateral whose vertices are $(0, 0)$, $(4, 5)$, $(9, 21)$, $(-3, 7)$.

2001.4 E is a point in the interior of rectangle $ABCD$. $AB = 6$, triangle ABE has area 6, and triangle CDE has area 12. Find $(EA)^2 - (EB)^2 + (EC)^2 - (ED)^2$.

2001.5 Two identical cones, each 2 inches in height, are held one directly above another with the pointed end facing down. The upper cone is completely filled with water. A small hole is punctured in the bottom of the upper cone so that the water trickles down into the bottom cone. When the water reaches a depth of 1 inch in the bottom cone, what is its depth in the upper cone?

2001.6 Find the radius of a circle inscribed in the triangle determined by the lines $4x + 3y = 24$, $56x - 33y = -264$, and $3x - 4y = 18$.

2001.7 In the figure, AB is tangent at A to the circle with center O, point D is interior to the circle, and DB intersects the circle at C. If $BC = DC = 3$, $OD = 2$ and $AB = 6$, then find the radius of the circle.
`https://cdn.artofproblemsolving.com/attachments/2/2/4275723bedf7569320bac52fc56270bb34bf9`
`png`

2001.8 Let S be the solid tetrahedron with boundary points $(0, 0, 0)$, $(2, 4, 0)$, $(5, 1, 0)$, $(3, 2, 10)$. Let $z_1 = \max\left\{q \middle| \left(\frac{12}{5}, \frac{23}{10}, q\right) \in S\right\}$ and let $z_2 = \max\left\{r \middle| \left(\frac{9}{2}, \frac{5}{4}, r\right) \in S\right\}$. Find $z_1 - z_2$.

2001.9 Circles A and B are tangent and have radii 1 and 2, respectively. A tangent to circle A from the point B intersects circle A at C. D is chosen on circle B so that AC is parallel to BD and the two segments BC and AD do not intersect. Segment AD intersects circle A at E. The line through B and E intersects circle A through another point F. Find EF.

2001.10 E is a point inside square $ABCD$ such that $\angle ECD = \angle EDC = 15^o$. Find $\angle AEB$.

2002.1 In the figure below, what is the length of x?
https://cdn.artofproblemsolving.com/attachments/7/7/655ad482e57c1231d21f0c4813ba8ddcfa8d3
png

2002.2 Upon cutting a certain rectangle in half, you obtain two rectangles that are scaled down versions of the original. What is the ratio of the longer side length to the shorter side length?

2002.3 An equilateral triangle has sides 1 inch long. An ant walks around the triangle maintaining a distance of 1 inch from the triangle at all times. How far does the ant walk?

2002.4 Given that the circles below each have radius 1 and are pairwise tangent, what is the area of the shaded region?
https://cdn.artofproblemsolving.com/attachments/6/6/4d813e0d6fd3d30f0e619e81ebedb2499c365
png

2002.5 A lattice point is a point on the coordinate plane with integer values for x and y. How many lattice points lie on a circle centered at the origin with radius 25?

2002.6 A *kite* is a quadrilateral with sides of length a, a, b, and b, where the sides of length a are adjacent, as are the sides of length b. If such a kite can be inscribed in a circle, then what is its area?

2002.7 Consider a regular n-gon with side length s. What is the ratio of the area to the square of the perimeter in terms of n and s? (expressed in fraction form)

2002.8 Four regular triangular pyramids are in a line. The first three have side lengths of 3, 4, and 5, and the volume of the last pyramid is the sum of the volumes of the first three. What is the side length of the last pyramid?

2002.9 A ladder of height h leans against a wall. It starts out flat against the wall, and then the base slides out along the ground until the ladder lies flat. The ladder touches the wall throughout this motion. What is the area underneath the path traced by the midpoint of the ladder?

2002.10 In the diagrams below, each circle is inscribed in the surrounding square, and each square is inscribed in the surrounding circle. Suppose the pattern continues on to infinity. If the outermost square has side length 1, what will the area of the shaded region be?
https://cdn.artofproblemsolving.com/attachments/b/f/f9c82a67b27ba5010f5fd7efdfaff3249045
png

2003.1 $ABCD$ is a square with sides of length 1. Suppose that a point E is placed somewhere on the edge CD. Let M be the maximum possible area of $\triangle ABE$, and let m be the minimum possible area of $\triangle ABE$. What is m/M ?

2003.2 Points A, B and C are such so that AC and BC define unique lines parallel to each other. If $A = (1, 2)$ and $B = (5, 6)$, and $AC = 3BC$, then what are the possible locations for C?

2003.3 In the diagram below, $\overline{BC} \perp \overline{AD}$, $AC = 6$, $CD = \frac{27}{2}$ and $m\angle BAC = m\angle CBD$. Find the length of BD.
https://cdn.artofproblemsolving.com/attachments/7/4/96bf0cb78f9a5047a4fd258b736dba59bdaf0
png

2003.4 Patty and Selma are racing through a park. The park has two concentric circular paths joined by two radial paths, one of which hits the outer circle at the point where they enter the park. The exit is at the intersection of the other radial path and the outer circle. Patty follows the radial path to the inner circle, walks around the short way to the other radial path and down it to the exit. Selma just walks the short way around the outer circle to the exit. They move at the same rate and meet up at the exit at the same time. What is the smaller angle made by the two radial paths?

2003.5 Circle O is inscribed in $\triangle ABC$ and has radius 1. $\triangle ABC$ is isosceles with $AC = BC$. Suppose that $AB = 2\sqrt{3}$. Find the area of the shaded region.
https://cdn.artofproblemsolving.com/attachments/d/4/f76f7af607f7be9db0899c4aa8a72d4952aaa
png

2003.6 In trapezoid $ABCD$ with $AB||CD$, $AB = 20$, $CD = 3$, $\angle ABC = 32^o$ and $\angle BAD = 58^o$. Compute the distance from the midpoint of AB to the midpoint of CD.

2003.7 Two spherical balls lie on the ground touching. If one ball has a radius of 8 inches and the point of contact is 10 inches above the ground, what is the radius of the other ball?

2003.8 In circle O, $OA \perp OB$ and $OB \perp CD$. CD has length $\sqrt{3}$ and arcAC has length 6. $\angle AOC = 120^o$. FindDB.
https://cdn.artofproblemsolving.com/attachments/d/d/3981be2c382fb0343a62218e72dc14d779f06
png

2003.9 A circular pizza of diameter 16 is cut so that two perpendicular diameters are each divided into 4 equal lengths. Find the area of the shaded corner piece.
https://cdn.artofproblemsolving.com/attachments/a/2/77fe364ff22c34af99f18d50620589ad9e9e5
png

2003.10 Let the area of the first figure below, the solid equilateral triangle, be 1. Then suppose that smaller equilateral triangles are successively removed step by step in the manner depicted below. What is the perimeter of the shaded region of the nth figure?
https://cdn.artofproblemsolving.com/attachments/9/5/2a5a10b8aece770776220b30a3e63b05d61a2
png

2003.R11 Consider 3 pairwise tangent circles with centers A, B, C. Given that $BC = 1$, $m\angle A = 45^o$, $m\angle B = 60^o$, $m\angle C = 75^o$, what is the area of $\triangle\ ABC$ that is not inside of any of the three circles?

2004.1 In the diagram below, the outer circle has radius 3, and the inner circle has radius 2. What is the area of shaded region?
https://cdn.artofproblemsolving.com/attachments/9/1/83314a8220381f48ec1718991e64af93e31ec
png

2004.2 A parallelogram has 3 of its vertices at $(1, 2)$, $(3, 8)$, and $(4, 1)$. Compute the sum of the possible x-coordinates for the 4th vertex.

2004.3 AC is 2004. CD bisects angle C. If the perimeter of ABC is 6012, find $(AC \times BC)/(AD \times BD)$.
https://cdn.artofproblemsolving.com/attachments/8/3/5e5db364214eb9bea060d16e924d0c03976e1
png

2004.4 P is inside rectangle $ABCD$. $PA = 2$, $PB = 3$, and $PC = 10$. Find PD.

2004.5 Find the area of the region of the xy-plane defined by the inequality $|x| + |y| + |x + y| \le 1$.

2004.6 We inscribe a square in a circle of radius 1 and shade the region between them. Then we incribe another circle in the square and another square in the new circle and shade the region between the new circle and square. After we have repeated this process infinitely many times, what is the area of the shaded region?
https://cdn.artofproblemsolving.com/attachments/4/6/895c108d532a3ad97470176a66736bbdef539
png

2004.7 Yet another trapezoid $ABCD$ has AD parallel to BC. AC and BD intersect at P. If $[ADP]/[BCP] = 1/2$, find $[ADP]/[ABCD]$.

(Here the notation $[P_1 ... P_n]$ denotes the area of the polygon $P_1 ... P_n$.)

2004.8 A triangle has side lengths 18, 24, and 30. Find the area of the triangle whose vertices are the incenter, circumcenter, and centroid of the original triangle.

2004.9 Given is a regular tetrahedron of volume 1. We obtain a second regular tetrahedron by reflecting the given one through its center. What is the volume of their intersection?

2004.10 Right triangle XYZ has right angle at Y and $XY = 228$, $YZ = 2004$. Angle Y is trisected, and the angle trisectors intersect XZ at P and Q so that X, P, Q, Z lie on XZ in that order. Find the value of $(PY + YZ)(QY + XY)$.

2005.1 A dog is tied via a 30 ft. leash to one corner of a 10 ft. by 20 ft. dog pen. Given that the dog is initially on the outside of the pen and that neither he (nor his leash) can cross the pen's fence, what area of land does he have to roam in?

2005.2 Find the area of the shaded region:https://cdn.artofproblemsolving.com/attachments/a/a/61608242e7447a2fa3663cba2f61f4a310d08c.png

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2005.3 An infinite series of similar right triangles converges to point C. If $AE = 16$, and $ED = 8$, what is the sum of all the vertical segiments $(AE + BD + ...)$?
https://cdn.artofproblemsolving.com/attachments/a/9/27f13556933f5c04d85ca71eb47f4bb457cfe
png

2005.4 Find the center of the circle of radius 1 centered on the y-axis that is tangent to the parabola $y = x^2$ in two places.

2005.5 A regular hexagon is inscribed in a circle with radius r. Another regular hexagon is formed with 2 vertices on an edge of the first hexagon and 2 vertices on the circle as shown below. Find the ratio of the area of the smaller hexagon to the area of the larger hexagon.
https://cdn.artofproblemsolving.com/attachments/c/d/239e036f15b10247f1597c563928273bc12af
png

2005.6 In circle O, chords AB and AC are drawn so that $AB = AC$. Chord MN is drawn intersecting AB and AC at points R and S, respectively. Given that $AR = SC = 17$ and $RB = AS = 13$, what is the maximum value of $|MR - NS|$?

2005.7 Suppose $\triangle ABC$ is an equilateral triangle with area 1. Points Q and P are on AB and points R and S are on AC with QR, PS and BC all parallel to each other. Also, $\overline{QR} < \overline{PS}$. Points P' and S' are chosen on BC so that PP' and SS' are each perpendicular to BC. Likewise, Q' and R' are chosen on PS so that QQ' and RR' are perpendicular to PS. What is the maximum possible area enclosed by the union of the two rectangles $PP'S'S$ and $QQ'R'R$?

2005.8 A unit lattice square in the plane is a square of side 1 whose vertices have integer coordinates. Given that $(20.05)^2 \cdot \pi \sim 1260$. Let N be the number of lattice squares that are entirely contained in a circle of radius 20.05 centered at the origin. Find $\left\lfloor \frac{N}{10} \right\rfloor$, where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

(Hint: A very good approximation for area in the plane of a smooth figure is given by $I + \frac{E}{2}$ where $I$is the number of unit lattice squares contained in the figure and E is the number intersected by the boundary)

2005.9 For each θ created by line OA and the x-axis, point X is the point (-2θ) from $\overrightarrow{AB}$, where $OA = 6$, and $AX = 3$. Find the area enclosed by X as A takes each point along the unit circle.

2005.10 An internal diagonal of a rectangular prism connects 2 vertices and does not lie entirely in one face. Let P be a rectangular prism with volume 1, and let l be one of its internal diagonals.

Suppose C is a vertex of the prism that is not one of the vertices of ℓ. A point K is chosen on ℓ, and a new prism p' is formed such that $\overline{CK}$ is an internal diagonal of p', and the faces of p' are parallel to those of p. What is the maximum volume of p'?

2006.1 Given a cube, determine the ratio of the volume of the octahedron formed by connecting the centers of each face of the cube to the volume of the cube

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2006.2 Given square $ABCD$ of side length 1, with E on $\overline{CD}$ and F in the interior of the square so that $\overline{EF} \perp \overline{DC}$ and $\overline{AF} \cong \overline{BF} \cong \overline{EF}$, find the area of the quadrilateral $ADEF$.
https://cdn.artofproblemsolving.com/attachments/3/2/a50ac80bbe4c97cc116236b3c071622fab013
png

2006.3 Circle γ is centered at $(0, 3)$ with radius 1. Circle δ is externally tangent to circle γ and tangent to the x-axis. Find an equation, solved for y if possible, for the locus of possible centers (x, y) of circle δ.

2006.4 The distance AB is ℓ. Find the area of the locus of points X such that $15^o \leq \angle AXB \leq 30^o$ and X is on the same side of line AB as a given point C.

2006.5 Let S denote a set of points (x, y, z). We define the shadow of S to be the set of points (x, y) for which there exists a real number z such that (x, y, z) is in S. For example, the shadow of a sphere with radius r centered on the z axis is a circle in the xy plane centered at the origin with radius r. Suppose a cube has a shadow consisting of a regular hexagon witih area $147\sqrt{3}$. What is the side length of the cube?

2006.6 A circle of radius R is placed tangent to two perpendicular lines. Another circle is placed tangent to the same two lines and the first circle. In terms of R, what is the radius of a third circle that is tangent to one line and tangent to both other circles

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2006.7 A certain $2'$ by $1'$ pool table has pockets, denoted $[A, ..., F]$ as shown. A pool player strikes a ball at point x, $\frac{1}{4}$ of the way up side $\overline{AC}$, aiming for a point $1.6'$ up the opposite side of the table. He makes his mark, and the ball ricochets around the edges of the table until it finally lands in one of the pockets. How many times does it ricochet before it falls into a pocket, and which pocket? Write your answer in the form $\{C, 2006\}$.
https://cdn.artofproblemsolving.com/attachments/c/c/72be287af1dc9c7ba2822d901e4fc82159c7c
png

2006.8 In triangle $\triangle PQR$, the altitudes from P, Q and R measure 5, 4 and 4, respectively. Find $\overline{QR}^2$.

2006.9 Poles A, B, and $P_1, P_2, P_3, ...$ are vertical line segments with bases on the x-axis. The tops of poles A and B are $(0, 1)$ and $(200, 5)$, respectively. A string S connects $(0, 1)$ and $(200, 0)$ and intersects another string connecting $(0, 0)$ and $(200, 5)$ at point T. Pole P_1 is constructed with T as its top point. For each integer $i > 1$, pole P_i is constructed so that its top point is the intersection of S and the line segment connecting the base of P_{i-1} (on the x-axis) and the top of pole B. Find the height of pole P_{100}.

2006.10 In triangle $\triangle ABC$, points P, Q and R lie on sides AB, BC and AC, respectively, so that $\frac{AP}{PB} = \frac{BQ}{QC} = \frac{CR}{RA} = \frac{1}{3}$. If the area of $\triangle ABC$ is 1, determine the area of the triangle formed by the points of intersection of lines AQ, BR and CP.

2007.1 An equilateral triangle has perimeter numerically equal to its area, which is not zero. Find its side length.

2007.2 Two spheres of radius 2 pass through each other's center. find the surface area of the regular octahedron inscribed within the space enclosed by both spheres.

2007.3 Cumulation of a polyhedron means replacing each face with a pyramid of height h using the face as a base. There is a cumulation of the cube of side length s which (after removing uncessary edges) has twelve sides, each a congruent rhombus. what is the height h used in this cumulation?

2007.4 Nathan is standing on vertex A of triangle ABC, with $AB = 3$, $BC = 5$, and $CA = 4$. Nathan walks according to the following plan: He moves along the altitude-to-the-hypotenuse until he reaches the hypotenuse. He has now cut the original triangle into two triangles; he now walks along the altitude to the hypotenuse of the larger one. He repeats this process forever. What is the total distance that Nathan walks?

2007.5 Given an octahedron with every edge of length s, what is the radius of the largest sphere that it will fit in this octahedron?

2007.6 Let $TINA$ be a quadrilateral with $IA = 8, IN = 4, m\angle T = 30^\circ, m\angle NAT = 60^\circ$, and $m\angle TIA = m\angle INA$. Find NA.

2007.7 Two regular tetrahedra of side length 2 are positioned such that the midpoint of each side of one coincides with the midpoint of a side of the other, and the tetrahedra themselves do not coincide. Find the volume of the region in which they overlap.

2007.8 $\triangle ABC$ has $\overline{AB} = \overline{AC}$. Points M and N are midpoints of $\overline{AB}$ and $\overline{AC}$, respectively. The medians $\overline{MC}$ and $\overline{NB}$ intersect at a right angle. Find $(\frac{\overline{AB}}{\overline{BC}})^2$.

2007.9 Points P, Q, R, S, T lie in the plane with S on $\overline{PR}$ and R on $\overline{QT}$. If $PQ = 5, PS = 3, PR = 5, QS = 3$, and $RT = 4/\sqrt{3}$, what is ST?

2007.10 A car starts moving at constant speed at the origin facing in the positive y-direction. Its minimum turning radius is such that it the soonest it can return to the x-axis is after driving a distance

d. Let Γ be the boundary of the region the car can reach by driving at most a distance d; find an $x > 0$ so that $\left(x, \frac{d}{3} + \frac{d\sqrt{3}}{2\pi}\right)$ is on Γ.

2008.1 A regular polygon of side length 1 has the property that if regular pentagons of side length 1 are placed on each side, then each pentagon shares a side with the two adjacent ones. How many sides does such a polygon have?

2008.2 John stands against one wall of a square room with walls of length 4 meters each. He kicks a frictionless, perfectly elastic ball in such a way that it bounces off the three other walls once each and returns to him (diagram not geometrically accurate). How many meters does the ball travel?
https://cdn.artofproblemsolving.com/attachments/9/c/71e3037ea7d49da0c1c235b4c44f6eae4b053
png

2008.3 A cube is inscribed in a sphere of radius r. Find the ratio of the volume of the cube to that of the sphere.

2008.4 A circle of radius 144 has three smaller circles inside it, all congruent. Each small circle is tangent to the other two and to the large circle. Find the radius of one of the smaller circles.

2008.5 In $\triangle ABC$, $\angle C$ is right, $AC = 2 - \sqrt{3} + x$ and $BC = 1 - 2x + x\sqrt{3}$. Find $m\angle B$.

2008.6 Points A, B, C lie on sides $\overline{DE}$, $\overline{EF}$. and FE of $\triangle\ DEF$, respectively. If $DA = 3$, $AE = 2$, $EB = 2$, $BF = 11$, $FC = 11$, and $CD = 1$, find the area of $\triangle\ ABC$.

2008.7 What is the area of the incircle of a triangle with side lengths 10040, 6024, and 8032?

2008.8 Rhombus $ABCD$ has side length ℓ, with $\cos(m\angle B) = -\frac{2}{3}$. The circle through points A, B, and D has radius 1. Find ℓ.

2008.9 A trapezoid has bases of length 10 and 15. Find the length of the segment that stretches from one leg of the trapezoid to the other, parallel to the bases, through the intersection point of the diagonals.

2008.10 A regular polygon with 40 sides, all of length 1, is divided into triangles, with each vertex of each triangle being a vertex of the original polygon. Let A be the area of the smallest triangle. What is the minimum number of square root signs needed to express the exact value of A?

2009.1 The sum of all of the interior angles of seven polygons is 180×17. Find the total number of sides of the polygons.

2009.2 The pattern in the figure below continues inward infinitely. The base of the biggest triangle is 1. All triangles are equilateral. Find the shaded area.
https://cdn.artofproblemsolving.com/attachments/e/5/8d1a2a072af330a7642f4fbd88d737a1ac296
png

2009.3 Given a regular pentagon, fi nd the ratio of its diagonal, d, to its side, a

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2009.4 $ABCD$ forms a rhombus. E is the intersection of AC and BD. F lie on AD such that EF is perpendicular to FD. Given $EF = 2$ and $FD = 1$. Find the area of the rhombus $ABCD$

2009.5 In the 2009 Stanford Olympics, Willy and Sammy are two bikers. The circular race track has two
lanes, the inner lane with radius 11, and the outer with radius 12. Willy will start on the inner lane, and Sammy on the outer. They will race for one complete lap, measured by the inner track.
What is the square of the distance between Willy and Sammy's starting positions so that they will both race
the same distance? Assume that they are of point size and ride perfectly along their respective lanes

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2009.6 Equilateral triangle ABC has side lengths of 24. Points D, E, and F lies on sides BC, CA, AB such that $AD \perp BC$, $DE \perp AC$, and $EF \perp AB$. G is the intersection of AD and EF. Find the area of quadrilateral $BFGD$

2009.7 Four disks with disjoint interiors are mutually tangent. Three of them are equal in size and the fourth one is smaller. Find the ratio of the radius of the smaller disk to one of the larger disks.

2009.8 Three points are randomly placed on a circle. What is the probability that they lie on the same semicircle

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2009.9 Two circles with centers A and B intersect at points X and Y. The minor arc $\angle XY = 120$ degrees with respect to circle A, and $\angle XY = 60$ degrees with respect to circle B. If $XY = 2$, find the area shared by the two circles.

2009.10 Right triangle ABC is inscribed in circle W. $\angle CAB = 65$ degrees, and $\angle CBA = 25$ degrees. The median from C to AB intersects W and line D. Line l_1 is drawn tangent to W at A. Line l_2 is drawn tangent to W at D. The lines l_1 and l_2 intersect at P
Determine $\angle APD$

2010.1 Find the reflection of the point $(11, 16, 22)$ across the plane $3x + 4y + 5z = 7$.

2010.2 Find the radius of a circle inscribed in a triangle with side lengths 4, 5, and 6

2010.3 Find the volume of a regular cubeoctahedron of side length 1. This is a solid whose faces comprise 6 squares and 8 equilateral triangles, arranged as in the diagram below.
https://cdn.artofproblemsolving.com/attachments/2/7/e758708c98ad5d8cd526ff0579d9eba6f9b8e
png

2010.4 Given triangle ABC. D lies on BC such that AD bisects BAC. Given $AB = 3$, $AC = 9$, and $BC = 8$. Find AD.

2010.5 Find the sum of angles $A, B, C, D, E, F, G, H, I$ in the following diagram:
https://cdn.artofproblemsolving.com/attachments/c/2/26f04a7c3c97ae5a995ca8e20d599c399b5c8
png

2010.6 In the diagram below, let $OT = 25$ and $AM = MB = 30$. Find MD
https://cdn.artofproblemsolving.com/attachments/c/6/e4acdf3fe5ec81b7747b800ffc33f87d0a961
png

2010.7 ABC is a triangle with $AB = 5$, $BC = 6$, and $CA = 7$. Squares are drawn on each side, as in the image below. Find the area of hexagon $DEFGHI$.
https://cdn.artofproblemsolving.com/attachments/3/d/81791cf692dfdbb661c4810beba0e93264056
png

2010.8 A sphere of radius 1 is internally tangent to all four faces of a regular tetrahedron. Find the tetrahedron's volume.

2010.9 For an acute triangle ABC and a point X satisfying $\angle ABX + \angle ACX = \angle CBX + \angle BCX$.

Fi nd the minimum length of AX if $AB = 13$, $BC = 14$, and $CA = 15$.

2010.10 A, B, C, D are points along a circle, in that order. AC intersects BD at X. If $BC = 6$, $BX = 4$, $XD = 5$, and $AC = 11$, fi nd AB

2010.R11 Given the three points $(1608, 2010, 2010)$, $(2010, 2412, 2010)$, and $(2010, 2010, 2412)$. Find the area of the circle defined by these three points.

2010.R12 Suppose we have a polyhedron consisting of triangles and quadrilaterals, and each vertex is shared by exactly 4 triangles and one quadrilateral. How many vertices are there?

2010.R13 Given the following circular section, write the height h, the height of the circle above the x-axis at a given x, as a function of x, with $-R \leq x \leq R$.

(Note θ and R are constants and θ is the angle between the x-axis and the tangent line to the circle at $x = -R$.)

https://cdn.artofproblemsolving.com/attachments/8/a/b41381338ba08f84046eaa5f84b584ae42f56
png

2010.R14 We are given a coin of diameter $\frac{1}{2}$ and a checkerboard of 1×1 squares of area 2010×2010. We must toss the coin such that it lands completely on the checkerboard. If the probability that the coin doesn't touch any of the lattice lines is $\frac{a^2}{b^2}$ where $\frac{a}{b}$ is a reduced fraction, find $a+b$.

2011.1 Triangle ABC has side lengths $BC = 3$, $AC = 4$, $AB = 5$. Let P be a point inside or on triangle ABC and let the lengths of the perpendiculars from P to BC, AC, AB be D_a, D_b, D_c respectively. Compute the minimum of $D_a + D_b + D_c$.

2011.2 Pentagon $ABCDE$ is inscribed in a circle of radius 1. If $\angle DEA \cong \angle EAB \cong \angle ABC$, $m\angle CAD = 60^o$, and $BC = 2DE$, compute the area of $ABCDE$.

2011.3 Let circle O have radius 5 with diameter $\overline{AE}$. Point F is outside circle O such that lines $\overline{FA}$ and $\overline{FE}$ intersect circle O at points B and D, respectively. If $FA = 10$ and $m\angle FAE = 30^o$, then the perimeter of quadrilateral ABDE can be expressed as $a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6}$, where a, b, c, and d are rational. Find $a + b + c + d$.

2011.4 Let ABC be any triangle, and D, E, F be points on $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ such that $CD = 2BD$, $AE = 2CE$ and $BF = 2AF$. $\overline{AD}$ and $\overline{BE}$ intersect at X, $\overline{BE}$ and $\overline{CF}$ intersect at Y , and $\overline{CF}$ and $\overline{AD}$ intersect at Z. Find $\frac{Area\,(\triangle ABC)}{Area\,(\triangle XYZ)}$.

2011.5 Let $ABCD$ be a cyclic quadrilateral with $AB = 6$, $BC = 12$, $CD = 3$, and $DA = 6$. Let E, F be the intersection of lines AB and CD, lines AD and BC respectively. Find EF.

2011.6 Two parallel lines ℓ_1 and ℓ_2 lie on a plane, distance d apart. On ℓ_1 there are an infinite number of points $A_1, A_2, A_3, \dots$, in that order, with $A_nA_{n+1} = 2$ for all n. On ℓ_2 there are an infinite number of points $B_1, B_2, B_3, \dots$, in that order and in the same direction, satisfying $B_nB_{n+1} = 1$ for all n. Given that A_1B_1 is perpendicular to both ℓ_1 and ℓ_2, express the sum $\sum_{i=1}^{\infty} \angle A_iB_iA_{i+1}$ in terms of d.

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`png`

2011.7 In a unit square $ABCD$, find the minimum of $\sqrt{2}AP + BP + CP$ where P is a point inside $ABCD$.

2011.8 We have a unit cube $ABCDEFGH$ where $ABCD$ is the top side and $EFGH$ is the bottom side with E below A, F below B, and so on. Equilateral triangle BDG cuts out a circle from the cube's inscribed sphere. Find the area of the circle.

2011.9 We have a circle O with radius 10 and four smaller circles O_1, O_2, O_3, $O_4$of radius 1 which are internally tangent to O, with their tangent points to O in counterclockwise order. The small circles do not intersect each other. Among the two common external tangents of O_1 and O_2, let

ℓ_{12} be the one which separates O_1 and O_2 from the other two circles, and let the intersections of ℓ_{12} and O be A_1 and B_2, with A_1 denoting the point closer to O_1. Define ℓ_{23}, ℓ_{34}, ℓ_{41} and A_2, A_3, A_4, B_3, B_4, B_1 similarly. Suppose that the arcs A_1B_1, A_2B_2, and A_3B_3 have length π, $3\pi/2$, and $5\pi/2$ respectively. Find the arc length of A_4B_4.

2011.10 Given a triangle ABC with $BC = 5$, $AC = 7$, and $AB = 8$, find the side length of the largest equilateral triangle PQR such that A, B, C lie on QR, RP, PQ respectively.

2011.R11 Jeffrey starts out at $(0, 0)$ facing in some direction. Each second, Jeffrey walks forward 1 unit, and then turns counterclockwise by 45^o . When Jeffrey returns to his starting point, what is the area of the shape he has made?

2011.R12 Let $\triangle ABC$ be equilateral. Two points D and E are on side BC (with order B, D, E, C), and satisfy $\angle DAE = 30^o$. If $BD = 2$ and $CE = 3$, what is BC?
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2011.R13 Given $\triangle ABC$. Let A' lie on BC such that $BA' = \frac14 A'C$, B' lie on AC such that $AB' = B'C$, and C' lie on AB such that $2AC' = BC'$. Let D be the point of intersection between AA' and CC', E the intersection point of between AA' and BB' , and F the point of intersection between BB' and CC' . What is area $(\triangle DEF)$ /area $(\triangle ABC)$?
