Geometry Rounds from Stanford and Rice Math Tournament, many years they shared problems, T= Tiebreaker, 2015-16 only RMT, 2018-21 only SMT

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2012.1 A circle with radius 1 has diameter AB. C lies on this circle such that ratio of lengths of arcs $AC/BC = 4$. $\overline{AC}$ divides the circle into two parts, and we will label the smaller part Region I. Similarly, $\overline{BC}$ also divides the circle into two parts, and we will denote the smaller one as Region II. Find the positive difference between the areas of Regions I and II.

2012.2 In trapezoid $ABCD$, $BC \parallel AD$, $AB = 13$, $BC = 15$, $CD = 14$, and $DA = 30$. Find the area of $ABCD$.

2012.3 Let ABC be an equilateral triangle with side length 1. Draw three circles O_a, O_b, and O_c with diameters BC, CA, and AB, respectively. Let S_a denote the area of the region inside O_a and outside of O_b and O_c. Define S_b and S_c similarly, and let S be the area of the region inside all three circles. Find $S_a + S_b + S_c - S$.

2012.4 Let $ABCD$ be a rectangle with area 2012. There exist points E on AB and F on CD such that $DE = EF = FB$. Diagonal AC intersects DE at X and EF at Y . Compute the area of triangle EXY .

2012.5 What is the radius of the largest sphere that fits inside an octahedron of side length 1?

2012.6 A red unit cube $ABCDEFGH$ (with E below A, F below B, etc.) is pushed into the corner of a room with vertex E not visible, so that faces $ABFE$ and $ADHE$ are adjacent to the wall and face $EFGH$ is adjacent to the floor. A string of length 2 is dipped in black paint, and one of its endpoints is attached to vertex A. How much surface area on the three visible faces of the cube can be painted black by sweeping the string over it?

2012.7 Let ABC be a triangle with incircle O and side lengths $5, 8$, and 9. Consider the other tangent line to O parallel to BC, which intersects AB at B_a and AC at C_a. Let r_a be the inradius of triangle AB_aC_a, and define r_b and r_c similarly. Find $r_a + r_b + r_c$.

2012.8 Let ABC be a triangle with side lengths 5, 6, and 7. Choose a radius r and three points outside the triangle O_a, O_b, and O_c, and draw three circles with radius r centered at these three points. If circles O_a and O_b intersect at C, O_b and O_c intersect at A, O_c and O_a intersect at B, and all three circles intersect at a fourth point, find r.

2012.9 In quadrilateral $ABCD$, $m\angle ABD \cong, m\angle BCD$ and $\angle ADB = \angle ABD + \angle BDC$. If $AB = 8$ and $AD = 5$, find BC.

2012.10 A large flat plate of glass is suspended $\sqrt{2/3}$ units above a large flat plate of wood. (The glass is infinitely thin and causes no funny refractive effects.) A point source of light is suspended $\sqrt{6}$ units above the glass plate. An object rests on the glass plate of the following description. Its base is an isosceles trapezoid $ABCD$ with $AB \parallel DC$, $AB = AD = BC = 1$, and $DC = 2$. The point source of light is directly above the midpoint of CD. The object's upper face is a triangle EFG with $EF = 2$, $EG = FG = \sqrt{3}$. G and AB lie on opposite sides of the rectangle $EFCD$. The other sides of the object are $EA = ED = 1$, $FB = FC = 1$, and $GD = GC = 2$. Compute the area of the shadow that the object casts on the wood plate.

2012.T1 In $\triangle\ ABC$, the altitude to $\overline{AB}$ from C partitions $\triangle\ ABC$ into two smaller triangles, each of which is similar to $\triangle\ ABC$. If the side lengths of $\triangle\ ABC$ and of both smaller triangles are all integers, find the smallest possible value of AB.

2012.T2 Four points O, A, B, and C satisfy $OA = OB = OC = 1$, $\angle AOB = 60^o$, and $\angle BOC = 90^o$. B is between A and C (i.e. $\angle AOC$ is obtuse). Draw three circles O_a, O_b, and O_c with diameters OA, OB, and OC, respectively. Find the area of region inside O_b but outside O_a and O_c.

2012.T3 Circles with centers O_1, O_2, and O_3 are externally tangent to each other and have radii 1, $\frac{1}{2}$, and $\frac{1}{4}$, respectively. Now for $i > 3$, let circle O_i be defined as the circle externally tangent tocircles O_{i-1} and O_{i-2} with radius 2^{1-i} that is farther from O_{i-3}. As n approaches infinity, thearea of triangle $O_1O_2O_n$ approaches the value A. Find A.

2013.1 In triangle ABC, $AC = 7$. D lies on AB such that $AD = BD = CD = 5$. Find BC.

2013.2 What is the perimeter of a rectangle of area 32 inscribed in a circle of radius 4?

2013.3 Robin has obtained a circular pizza with radius 2. However, being rebellious, instead of slicing the pizza radially, he decides to slice the pizza into 4 strips of equal width both vertically and horizontally. What is the area of the smallest piece of pizza?

2013.4 $ABCD$ is a regular tetrahedron with side length 1. Find the area of the cross section of $ABCD$ cut by the plane that passes through the midpoints of AB, AC, and CD.

2013.5 In square $ABCD$ with side length 2, let P and Q both be on side AB such that $AP = BQ = \frac{1}{2}$. Let E be a point on the edge of the square that maximizes the angle PEQ. Find the area of triangle PEQ.

2013.6 $ABCD$ is a rectangle with $AB = CD = 2$. A circle centered at O is tangent to BC, CD, and AD (and hence has radius 1). Another circle, centered at P, is tangent to circle O at point T and is also tangent to AB and BC. If line AT is tangent to both circles at T, find the radius of circle P.

2013.7 $ABCD$ is a square such that AB lies on the line $y = x + 4$ and points C and D lie on the graph of parabola $y^2 = x$. Compute the sum of all possible areas of $ABCD$.

2013.8 Let equilateral triangle ABC with side length 6 be inscribed in a circle and let P be on arc AC such that $AP \cdot PC = 10$. Find the length of BP.

2013.9 In tetrahedron $ABCD$, $AB = 4$, $CD = 7$, and $AC = AD = BC = BD = 5$. Let I_A, I_B, I_C, and I_D denote the incenters of the faces opposite vertices A, B, C, and D, respectively. It is provable that AI_A intersects BI_B at a point X, and CI_C intersects DI_D at a point Y. Compute XY.

2013.10 Let triangle ABC have side lengths $AB = 16, BC = 20, AC = 26$. Let $ACDE, ABFG$, and $BCHI$ be squares that are entirely outside of triangle ABC. Let J be the midpoint of EH, K be the midpoint of DG, and L be the midpoint of AC. Find the area of triangle JKL.

2013.T1 A circle of radius 2 is inscribed in equilateral triangle ABC. The altitude from A to BC intersects the circle at a point D not on BC. BD intersects the circle at a point E distinct from D. Find the length of BE.

2013.T2 Points A, B, and C lie on a circle of radius 5 such that $AB = 6$ and $AC = 8$. Find the smaller of the two possible values of BC.

2013.T3 In quadrilateral $ABCD$, diagonals AC and BD intersect at E. If $AB = BE = 5$, $EC = CD = 7$, and $BC = 11$, compute AE.

2014.1 The coordinates of three vertices of a parallelogram are $A(1, 1)$, $B(2, 4)$, and $C(-5, 1)$. Compute the area of the parallelogram.

2014.2 In a circle, chord AB has length 5 and chord AC has length 7. Arc AC is twice the length of arc AB, and both arcs have degree less than 180. Compute the area of the circle.

2014.3 Spencer eats ice cream in a right circular cone with an opening of radius 5 and a height of 10. If Spencer's ice cream scoops are always perfectly spherical, compute the radius of the largest scoop he can get such that at least half of the scoop is contained within the cone.

2014.4 Let ABC be a triangle such that $AB = 3$, $BC = 4$, and $AC = 5$. Let X be a point in the triangle. Compute the minimal possible value of $AX^2 + BX^2 + CX^2$

2014.5 Let ABC be a triangle where $\angle BAC = 30^\circ$. Construct D in $\triangle ABC$ such that $\angle ABD = \angle ACD = 30^\circ$. Let the circumcircle of $\triangle ABD$ intersect AC at X. Let the circumcircle of $\triangle ACD$ intersect AB at Y. Given that $DB - DC = 10$ and $BC = 20$, find $AX \cdot AY$.

2014.6 Let E be an ellipse with major axis length 4 and minor axis length 2. Inscribe an equilateral triangle ABC in E such that A lies on the minor axis and BC is parallel to the major axis. Compute the area of $\triangle ABC$.

2014.7 Let ABC be a triangle with $AB = 13$, $BC = 14$, and $AC = 15$. Let$D$ and E be the feet of the altitudes from A and B, respectively. Find the circumference of the circumcircle of $\triangle CDE$

2014.8 O is a circle with radius 1. A and B are fixed points on the circle such that $AB = \sqrt{2}$. Let C be any point on the circle, and let M and N be the midpoints of AC and BC, respectively. As C travels around circle O, find the area of the locus of points on MN.

2014.9 In cyclic quadrilateral $ABCD$, $AB = AD$. If $AC = 6$ and $\frac{AB}{BD} = \frac{3}{5}$, find the maximum possible area of $ABCD$.

2014.10 Let ABC be a triangle with $AB = 12$, $BC = 5$, $AC = 13$. Let$D$ and E be the feet of the internal and external angle bisectors from B, respectively. (The external angle bisector from B bisects the angle between BC and the extension of AB.) Let ω be the circumcircle of $\triangle BDE$, extend AB so that it intersects ω again at F. Extend FC to meet ω again at X, and extend AX to meet ω again at G. Find FG.

2014.T1 A square $ABCD$ with side length 1 is inscribed in a circle. A smaller square lies in the circle with two vertices lying on segment AB and the other two vertices lying on minor arc AB. Compute the area of the smaller square.

2014.T2 Let ABC be a triangle with sides $AB = 19$, $BC = 21$ and $AC = 20$. Let ω be the incircle of ABC with center I. Extend BI so that it intersects AC at E. If ω is tangent to AC at the point D, then find the length of DE.

2014.T3 Circle O has three chords, AD, DF, and EF. Point E lies along the arc AD. Point C is the intersection of chords AD and EF. Point B lies on segment AC such that $EB = EC = 8$. Given $AB = 6$, $BC = 10$, and $CD = 9$, find DF.
`https://cdn.artofproblemsolving.com/attachments/f/c/c36bff9ad04f13f7e227c57bddb53a0bfc056 png`

2015.1 Clyde is making a Pacman sticker to put on his laptop. A Pacman sticker is a circular sticker of radius 3 inches with a sector of 120^o cut out. What is the perimeter of the Pacman sticker in inches?

2015.2 In a certain right triangle, dropping an altitude to the hypotenuse divides the hypotenuse into two segments of length 2 and 3 respectively. What is the area of the triangle?

2015.3 Consider a triangular pyramid $ABCD$ with equilateral base ABC of side length 1. $AD = BD = CD$ and $\angle ADB = \angle BDC = \angle ADC = 90^o$. Find the volume of $ABCD$.

2015.4 Two circles with centers A and B respectively intersect at two points C and D. Given that A, B, C, D lie on a circle of radius 3 and circle A has radius 2, what is the radius of circle B?

2015.5 Consider two concentric circles of radius 1 and 2. Up to rotation, there are two distinct equilateral triangles with two vertices on the circle of radius 2 and the remaining vertex on the circle of radius 1. The larger of these triangles has sides of length a, and the smaller has sides of length b. Compute $a + b$.

2015.6 In a triangle ABC, let D and E trisect BC, so $BD = DE = EC$. Let F be the point on AB such that $\frac{AF}{FB} = 2$, and G on AC such that $\frac{AG}{GC} = \frac{1}{2}$. Let P be the intersection of DG and EF, and extend AP to intersect BC at a point X. Find $\frac{BX}{XC}$

2015.7 A unit sphere is centered at $(0, 0, 1)$. There is a point light source located at $(1, 0, 4)$ that sends out light uniformly in every direction but is blocked by the sphere. What is the area of the sphere's shadow on the $x - y$ plane? (A point (a, b, c) denotes the point in three dimensions with x-coordinate a, y-coordinate b, and z-coordinate c)

2015.8 Consider the parallelogram $ABCD$ such that $CD = 8$ and $BC = 14$. The diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E and $AC = 16$. Consider a point F on the segment $\overline{ED}$ with $FD = \frac{\sqrt{66}}{3}$. Compute CF.

2015.9 Triangle ABC is isoceles with $AB = AC = 2$ and $BC = 1$. Point D lies on AB such that the inradius of ADC equals the inradius of BDC. What is the inradius of ADC?

2015.10 For a positive real number k and an even integer $n \geq 4$, the k-*Perfect* n-gon is defined to be the equiangular n-gon $P_1P_2...P_n$ with $P_iP_{i+1} = P_{n/2+i}P_{n/2+i+1} = k^{i-1}$ for all $i \in \{1, 2, ..., n/2\}$, assuming the convention $P_{n+1} = P_1$ (i.e. the numbering wraps around). If $a(k, n)$ denotes the area of the k-Perfect n-gon, compute $\frac{a(2,24)}{a(4,12)}$.

2016.1 Form a triangle ABC with side lengths $AB = 12$, $AC = 8$, and $BC = 15$. Let the altitude from A to BC intersect BC at D and let AE be the angle bisector of $\angle BAC$, where E is on BC. Compute the length of DE.

2016.2 Consider a unit cube and a plane that slices through it. The plane passes through the midpoints of two adjacent edges on the top face, two on the bottom face, and the center of the cube. Compute the area of the cross section.

2016.3 Compute the area of the largest square that can be inscribed in a unit cube. You may assume that the square's vertces lie on the edges of the cube.

2016.4 Inside a circle of radius 1 are three circles of equal radius such that each of them is tangent to the other two and to the large circle. Determine the radius of one of the smaller circles.

2016.5 When circles of the same radius are packed into the plane with maximum density they form a regular lattice. Compute the packing density of this arrangement, that is, the fraction of area covered by circles.

2016.6 Consider a unit square $ABCD$. Let E be the midpoint of BC and F the intersection of AC and DE. Compute the area of triangle ADF.

2016.7 Consider a circular sector of unit radius and angle $arcsin\left(\frac{1}{3}\right)$. Let S be a square inscribed in the sector such that the axis of symmetry of the sector passes through the center of S, is parallel to two of the sides of S, and all four vertices of S are on the boundary of the sector. What is the area of S?

2016.8 Natasha walks along a closed convex polygonal curve of length 2016. She carries a paintbrush of length 1 and walking all the way around paints all the area as far as she can reach on the outside of the curve. What is that area?

2016.9 Four spheres of radius 1 are mutually tangent. What is the radius of the smallest sphere containing them?

2016.10 Consider a regular pentagon and connect each vertex to the pair of vertices farthest from it by line segments. The line segments intersect at 5 points to form another smaller pentagon. If the large pentagon has side length 1, compute the area of the smaller pentagon. Express your answer without trigonometric functions.

2018.1 Consider a semi-circle with diameter AB. Let points C and D be on diameter AB such that CD forms the base of a square inscribed in the semicircle. Given that $CD = 2$, compute the length of AB.

2018.2 Let $ABCD$ be a trapezoid with AB parallel to CD and perpendicular to BC. Let M be a point on BC such that $\angle AMB = \angle DMC$. If $AB = 3$, $BC = 24$, and $CD = 4$, what is the value of $AM + MD$?

2018.3 Let ABC be a triangle and D be a point such that A and D are on opposite sides of BC. Give that $\angle ACD = 75^o$, $AC = 2$, $BD = \sqrt{6}$, and AD is an angle bisector of both $\triangle\ ABC$ and $\triangle\ BCD$, find the area of quadrilateral $ABDC$.

2018.4 Let $a_1, a_2, ..., a_{12}$ be the vertices of a regular dodecagon D_1 (12-gon). The four vertices a_1, a_4, a_7, a_{10} form a square, as do the four vertices a_2, a_5, a_8, a_{11} and a_3, a_6, a_9, a_{12}. Let D_2 be the polygon formed by the intersection of these three squares. If we let$[A]$ denotes the area of polygon A, compute $\frac{[D_2]}{[D_1]}$.

2018,5 In $\triangle\ ABC$, $\angle ABC = 75^o$ and $\angle BAC$ is obtuse. Points D and E are on AC and BC, respectively, such that $\frac{AB}{BC} = \frac{DE}{EC}$ and $\angle DEC = \angle EDC$. Compute $\angle DEC$ in degrees.

2018.6 In $\triangle ABC$, $AB = 3$, $AC = 6$, and D is drawn on BC such that AD is the angle bisector of $\angle BAC$. D is reflected across AB to a point E, and suppose that AC and BE are parallel. Compute CE.

2018.7 Two equilateral triangles ABC and DEF, each with side length 1, are drawn in 2 parallel planes such that when one plane is projected onto the other, the vertices of the triangles form a regular hexagon $AFBDCE$. Line segments AE, AF, BF, BD, CD, and CE are drawn, and suppose that each of these segments also has length 1. Compute the volume of the resulting solid that is formed.

2018.8 Let ABC be a right triangle with $\angle ACB = 90^o$, $BC = 16$, and $AC = 12$. Let the angle bisectors of $\angle BAC$ and $\angle ABC$ intersect BC and AC at D and E respectively. Let AD and BE intersect at I, and let the circle centered at I passing through C intersect AB at P and Q such that $AQ < AP$. Compute the area of quadrilateral $DPQE$.

2018.9 Let $ABCD$ be a cyclic quadrilateral with $3AB = 2AD$ and $BC = CD$. The diagonals AC and BD intersect at point X. Let E be a point on AD such that $DE = AB$ and Y be the point of intersection of lines AC and BE. If the area of triangle ABY is 5, then what is the area of quadrilateral $DEYX$?

2018.10 Let ABC be a triangle with $AB = 13$, $AC = 14$, and $BC = 15$, and let Γ be its incircle with incenter I. Let D and E be the points of tangency between Γ and BC and AC respectively, and let ω be the circle inscribed in $CDIE$. If Q is the intersection point between Γ and ω and P is the intersection point between CQ and ω, compute the length of PQ.

2018.T1 Point E is on side CD of rectangle $ABCD$ such that $\frac{CE}{DE} = \frac{2}{5}$. If the area of triangle BCE is 30, what is the area of rectangle $ABCD$?

2018.T2 What is the largest possible height of a right cylinder with radius 3 that can fit in a cube with side length 12?

2018.T3 A triangle has side lengths of 7, 8, and 9. Find the radius of the largest possible semicircle inscribed in the triangle.

2019.1 Let $ABCD$ be a unit square. A semicircle with diameter AB is drawn so that it lies outside of the square. If E is the midpoint of arc AB of the semicircle, what is the area of triangle CDE

2019.2 A cat and mouse live on a house mapped out by the points $(-1, 0)$, $(-1, 2)$, $(0, 3)$, $(1, 2)$, $(1, 0)$. The cat starts at the top of the house (point $(0, 3)$) and the mouse starts at the origin (0, 0). Both start running clockwise around the house at the same time. If the cat runs at 12 units a minute and the mouse at 9 units a minute, how many laps around the house will the cat run before it catches the mouse?

2019.3 In triangle ABC with $AB = 10$, let$D$ be a point on side BC such that AD bisects $\angle BAC$. If $\frac{CD}{BD} = 2$ and the area of ABC is 50, compute the value of $\angle BAD$ in degrees.

2019.4 Let ω_1 and ω_2 be two circles intersecting at points P and Q. The tangent line closer to Q touches ω_1 and ω_2 at M and N respectively. If $PQ = 3$, $QN = 2$, and $MN = PN$, what is QM^2?

2019.5 The bases of a right hexagonal prism are regular hexagons of side length $s > 0$, and the prism has height h. The prism contains some water, and when it is placed on a flat surface with a hexagonal face on the bottom, the water has depth $\frac{s\sqrt{3}}{4}$. The water depth doesn't change when the prism is turned so that a rectangular face is on the bottom. Compute $\frac{h}{s}$.

2019.6 Let the altitude of $\triangle ABC$ from A intersect the circumcircle of $\triangle ABC$ at D. Let E be a point on line AD such that $E \neq A$ and $AD = DE$. If $AB = 13$, $BC = 14$, and $AC = 15$, what is the area of quadrilateral $BDCE$?

2019.7 Let G be the centroid of triangle ABC with $AB = 9$, $BC = 10$, and $AC = 17$. Denote D as the midpoint of BC. A line through G parallel to BC intersects AB at M and AC at N. If BG intersects CM at E and CG intersects BN at F, compute the area of triangle DEF.

2019.8 In the coordinate plane, a point A is chosen on the line $y = \frac{3}{2}x$ in the first quadrant. Two perpendicular lines ℓ_1 and ℓ_2 intersect at A where ℓ_1 has slope $m > 1$. Let ℓ_1 intersect the x-axis at B, and ℓ_2 intersects the x and y axes at C and D, respectively. Suppose that line BD has slope $-m$ and $BD = 2$. Compute the length of CD.

2019.9 Let $ABCD$ be a quadrilateral with $\angle ABC = \angle CDA = 45^o$, $AB = 7$, and $BD = 25$. If AC is perpendicular to CD, compute the length of BC.

2019.10 Let ABC be an acute triangle with $BC = 48$. Let M be the midpoint of BC, and let D and E be the feet of the altitudes drawn from B and C to AC and AB respectively. Let P be the intersection between the line through A parallel to BC and line DE. If $AP = 10$, compute the length of PM,

2019.T1 Let $ABCD$ be a quadrilateral with $\angle DAB = \angle ABC = 120^o$. If $AB = 3$, $BC = 2$, and $AD = 4$, what is the length of CD?

2019.T2 Let $ABCD$ be a rectangle with $AB = 8$ and $BC = 6$. Point E is outside of the rectangle such that $CE = DE$. Point D is reflected over line AE so that its image, D' , lies on the interior of the rectangle. Point D' is then reflected over diagonal AC, and its image lies on side AB. What is the length of DE?

2019.T3 Right triangle ABC with $\angle ABC = 90^o$ is inscribed in a circle ω_1 with radius 3. A circle ω_2 tangent to AB, BC, and ω_1 has radius 2. Compute the area of $\triangle\ ABC$.

2020.1 A circle with radius 1 is circumscribed by a rhombus. What is the minimum possible area of this rhombus?

2020.2 Let $\triangle\ ABC$ be a right triangle with $\angle ABC = 90^o$. Let the circle with diameter BC intersect AC at D. Let the tangent to this circle at D intersect AB at E. What is the value of $\frac{AE}{BE}$?

2020.3 Square $ABCD$ has side length 4. Points P and Q are located on sides BC and CD, respectively, such that $BP = DQ = 1$. Let AQ intersect DP at point X. Compute the area of triangle PQX.

2020.4 Let $ABCD$ be a quadrilateral such that $AB = BC = 13$, $CD = DA = 15$ and $AC = 24$. Let the midpoint of AC be E. What is the area of the quadrilateral formed by connecting the incenters of ABE, BCE, CDE, and DAE?

2020.5 Find the smallest possible number of edges in a convex polyhedron that has an odd number of edges in total has an even number of edges on each face.

2020.6 Consider triangle ABC on the coordinate plane with $A = (2, 3)$ and $C = \left(\frac{96}{13}, \frac{207}{13}\right)$. Let B be the point with the smallest possible y-coordinate such that $AB = 13$ and $BC = 15$. Compute the coordinates of the incenter of triangle ABC.

2020.7 Let ABC be an acute triangle with $BC = 4$ and $AC = 5$. Let D be the midpoint of BC, E be the foot of the altitude from B to AC, and F be the intersection of the angle bisector of $\angle BCA$ with segment AB. Given that AD, BE, and CF meet at a single point P, compute the area of triangle ABC. Express your answer as a common fraction in simplest radical form.

2020.8 Consider an acute angled triangle $\triangle\ ABC$ with side lengths 7, 8, and 9. Let H be the orthocenter of ABC. Let Γ_A, Γ_B, and Γ_C be the circumcircles of $\triangle\ BCH$, $\triangle\ CAH$, and $\triangle\ ABH$ respectively. Find the area of the region $\Gamma_A \cup \Gamma_B \cup \Gamma_C$ (the set of all points contained in at least one of Γ_A, Γ_B, and Γ_C).

2020.9 Let ABC be a right triangle with hypotenuse AC. Let G be the centroid of this triangle and suppose that we have $AG^2 + BG^2 + CG^2 = 156$. Find AC^2.

2020.10 Three circles with radii 23, 46, and 69 are tangent to each other as shown in the figure below (figure is not drawn to scale). Find the radius of the largest circle that can fit inside the shaded region.
`https://cdn.artofproblemsolving.com/attachments/6/d/158abc178e4ddd72541580958a4ee2348b202
png`

2020.T1 Pentagon $ABCDE$ has $AB = BC = CD = DE$, $\angle ABC = \angle BCD = 108^o$, and $\angle CDE = 168^o$. Find the measure of angle $\angle BEA$ in degrees.

2020.T2 On each edge of a regular tetrahedron, five points that separate the edge into six equal segments are marked. There are twenty planes that are parallel to a face of the tetrahedron and pass through exactly three of the marked points. When the tetrahedron is cut along each of these twenty planes, how many new tetrahedrons are produced?

2020.T3 Three cities that are located on the vertices of an equilateral triangle with side length 100 units. A missile flies in a straight line in the same plane as the equilateral triangle formed by the three citiies. The radar from City A reported that the closest approach of the missile was 20 units. The radar from City B reported that the closest approach of the missile was 60 units. However, the radar for city C malfunctioned and did not report a distance. Find the minimum possible distance for the closest approach of the missile to city C.

2021.1 A paper rectangle $ABCD$ has $AB = 8$ and $BC = 6$. After corner B is folded over diagonal AC, what is BD?

2021.2 Let $ABCD$ be a trapezoid with bases $AB = 50$ and $CD = 125$, and legs $AD = 45$ and $BC = 60$. Find the area of the intersection between the circle centered at B with radius BD and the circle centered at D with radius BD. Express your answer as a common fraction in simplest radical form and in terms of π.

2021.3 If r is a rational number, let $f(r) = \left(\frac{1-r^2}{1+r^2}, \frac{2r}{1+r^2}\right)$. Then the images of f forms a curve in the xy plane. If $f(1/3) = p_1$ and $f(2) = p_2$, what is the distance along the curve between p_1 and p_2?

2021.4 $\triangle\ A_0B_0C_0$ has side lengths $A_0B_0 = 13$, $B_0C_0 = 14$, and $C_0A_0 = 15$. $\triangle\ A_1B_1C_1$ is inscribed in the incircle of $\triangle\ A_0B_0C_0$ such that it is similar to the first triangle. Beginning with $\triangle\ A_1B_1C_1$, the same steps are repeated to construct $\triangle\ A_2B_2C_2$, and so on infinitely many times. What is the value of $\sum_{i=0}^{\infty} A_iB_i$?

2021.5 Let $ABCD$ be a square of side length 1, and let E and F be on the lines AB and AD, respectively, so that B lies between A and E, and D lies between A and F. Suppose that $\angle BCE = 20^o$ and $\angle DCF = 25^o$. Find the area of triangle $\triangle\ EAF$.

2021.6 $\odot A$, centered at point A, has radius 14 and $\odot B$, centered at point B, has radius 15. $AB = 13$. The circles intersect at points C and D. Let E be a point on $\odot A$, and F be the point where line EC intersects $\odot B$, again. Let the midpoints of DE and DF be M and N, respectively. Lines AM and BN intersect at point G. If point E is allowed to move freely on $\odot A$, what is the radius of the locus of G?

2021.7 An n-sided regular polygon with side length 1 is rotated by $\frac{180^o}{n}$ about its center. The intersection points of the original polygon and the rotated polygon are the vertices of a $2n$-sided regular polygon with side length $\frac{1-tan^2 10^o}{2}$. What is the value of n?

2021.8 In triangle $\triangle ABC$, $AB = 5$, $BC = 7$, and $CA = 8$. Let E and F be the feet of the altitudes from B and C, respectively, and let M be the midpoint of BC. The area of triangle MEF can be expressed as $\frac{a\sqrt{b}}{c}$ for positive integers a, b, and c such that the greatest common divisor of a and c is 1 and b is not divisible by the square of any prime. Compute $a + b + c$.

2021.9 Rectangle $ABCD$ has an area of 30. Four circles of radius $r_1 = 2$, $r_2 = 3$, $r_3 = 5$, and $r_4 = 4$ are centered on the four vertices A, B, C, and D respectively. Two pairs of external tangents are drawn for the circles at A and C and for the circles at B and D. These four tangents intersect to form a quadrilateral $WXYZ$ where $\overline{WX}$ and $\overline{YZ}$ lie on the tangents through the circles on A and C. If $\overline{WX} + \overline{YZ} = 20$, find the area of quadrilateral $WXYZ$.
`https://cdn.artofproblemsolving.com/attachments/5/a/cb3b3457f588a15ffb4c875b1646ef2aec8d1`
`png`

2021.10 In acute $\triangle ABC$, let points D, E, and F be the feet of the altitudes of the triangle from A, B,and C, respectively. The area of $\triangle AEF$ is 1, the area of $\triangle CDE$ is 2, and the area of $\triangle BFD$ is $2 - \sqrt{3}$. What is the area of $\triangle DEF$?

2021.T1 What is the radius of the largest circle centered at $(2, 2)$ that is completely bounded within the parabola $y = x^2 - 4x + 5$?

2021.T2 If two points are picked randomly on the perimeter of a square, what is the probability that the distance between those points is less than the side length of the square?

2021.T3 In quadrilateral $ABCD$, $CD = 14$, $\angle BAD = 105^o$, $\angle ACD = 35^o$, and $\angle ACB = 40^o$. Let the midpoint of CD be M. Points P and Q lie on $\overrightarrow{AM}$ and $\overrightarrow{BM}$, respectively, such that $\angle APB = 40^o$ and $\angle AQB = 40^o$. PB intersects CD at point R and QA intersects CD at point S. If $CR = 2$, what is the length of SM?
