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by parmenides51

–	level 2
1	In a 7×7 board, some squares are painted red. Let a be the number of rows that have an odd number of red squares and let b be the number of columns that have an odd number of red squares. Find all possible values of $a + b$. For each value found, give a example of how the board can be painted.
2	There are nine cards that have the digits $1, 2, 3, 4, 5, 6, 7, 8$ and 9 written on them, with one digit on each card. Using all the cards, some numbers are formed (for example, the numbers 8, 213, 94, 65 and 7). a) If all the numbers formed are prime, determine the smallest possible value of their sum. b) If all formed numbers are composite, determine the smallest possible value of their sum. Note: A number p is prime if its only divisors are 1 and p. A number is composite if it has more than two dividers. The number 1 is neither prime nor composite.
3	Let $ABCD$ be a square, E a point on the side CD, and F a point inside the square such that that triangle BFE is isosceles and $\angle BFE = 90^o$. If $DF = DE$, find the measure of angle $\angle FDE$.
4	a) A positive integer is written at each vertex of a triangle. Then on each side of the triangle the greatest common divisor of its ends is written. It is possible that the numbers written on the sides be three consecutive integers, in some order? b) A positive integer is written at each vertex of a tetrahedron. Then, on each edge of the tetrahedron is written the greatest common divisor of its ends . It is possible that the numbers written in the edges are six consecutive integers, in some order?
5	The vertices of a regular polygon with N sides are marked on the blackboard. Ana and Beto play alternately, Ana begins. Each player, in turn, must do the following: • join two vertices with a segment, without cutting another already marked segment; or • delete a vertex that does not belong to any marked segment. The player who cannot take any action on his turn loses the game. Determine which of the two players can guarantee victory: a) if $N = 28$ b) if $N = 29$
–	level 1

1 This morning, Emi dropped the watch and from there it started to move more slowly. When, according to the clock, 2 minutes have passed, in reality it has already been 3. Now it is $6:25$ pm and the clock says it is $3:30$ pm. What time did Emi drop the watch?

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2 Bob chose six of the nine digits from 1 to 9 and wrote the list, ordered from smallest to largest, of all three-digit numbers that can be formed using the digits you chose. At Bob's list, the number 317 appears at position 22. What number appears at position 60 in the list from Bob? Find all possibilities.

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3 Choose nine of the digits from 0 to 9 and place them in the boxes in the figure so that there are no repeated digits and the indicated sum is correct.
https://cdn.artofproblemsolving.com/attachments/6/2/7f06575ec70eb9ddd58c6cf9dd3cb60d306e7.png
Which digit was not used? You can fill in the boxes so that the unused digit is other?

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4 Ana and Bruno have an 8×8 checkered board. Ana paints each of the 64 squares with some color. Then Bruno chooses two rows and two columns on the board and looks at the 4 squares where they intersect. Bruno's goal is for these 4 squares to be the same color. How many colors, at least, must Ana use so that Bruno can't fulfill his objective? Show how you can paint the board with this amount of colors and explain because if you use less colors then Bruno can always fulfill his goal.

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5 Vero had an isosceles triangle made of paper. Using scissors, he divided it into three smaller triangles and painted them blue, red and green. Having done so, he observed that: • with the blue triangle and the red triangle an isosceles triangle can be formed, • with the blue triangle and the green triangle an isosceles triangle can be formed, • with the red triangle and the green triangle an isosceles triangle can be formed.
Show what Vero's triangle looked like and how he might have made the cuts to make this situation be possible.
