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by parmenides51

– level 2

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- 1** Let $ABCD$ be a rectangle. A line r moves parallel to AB and intersects diagonal AC , forming two triangles opposite the vertex, inside the rectangle. Prove that the sum of the areas of these triangles is minimal when r passes through the midpoint of segment AD .
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- 2** Joining $15^3 = 3375$ cubes of 1 cm^3 , bodies with a volume of 3375 cm^3 can be built. Indicate how two bodies A and B are constructed with 3375 cubes each and such that the lateral surface of B is 10 times the lateral surface of A .
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- 3** Natalia and Marcela count 1 by 1 starting together at 1, but Marcela's speed is triple that of Natalia (when Natalia says her second number, Marcela says the fourth number). When the difference of the numbers that they say in unison is any of the multiples of 29, between 500 and 600, Natalia continues counting normally and Marcela begins to count downwards in such a way that, at one point, the two say in unison the same number. What is said number?
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- 4** Let $ABCD$ be a square and let point F be any point on side BC . Let the line perpendicular to DF , that passes through B , intersect line DC at Q . What is value of $\angle FQC$?
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- 5** You have a 10×10 grid. A "move" on the grid consists of moving 7 squares to the right and 3 squares down. In case of exiting by a line, it continues at the beginning (left) of the same line and in case of ending a column, it continues at the beginning of the same column (above). Where should we start so that after 1996 moves we end up in a corner?

– level 1

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- 1** A terrain ($ABCD$) has a rectangular trapezoidal shape. The angle in A measures 90° . AB measures 30 m, AD measures 20 m and DC measures 45 m. This land must be divided into two areas of the same area, drawing a parallel to the AD side. At what distance from D do we have to draw the parallel?
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- 2** Considering the three-digit natural numbers, how many of them, when adding two of their digits, are double of their remainder? Justify your answer.
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- 3** A and B are two cylindrical containers that contain water. The height of the water at A is 1000 cm and at B , 350 cm. Using a pump, water is transferred from A to B . It is noted that, in container A ,

the height of the water decreases 4 cm per minute and in B it increases 9 cm per minute. After how much time, since the pump was started, will the heights at A and B be the same?

- 4 (a) In this drawing, there are three squares on each side of the square. Place a natural number in each of the boxes so that the sum of the numbers of two adjacent boxes is always odd.

<https://cdn.artofproblemsolving.com/attachments/e/6/75517b7d49857abd3f8f0430a70ae5b0eb155.gif>

- (b) In this drawing, there are now four squares on each side of the triangle. Justify why a natural number cannot be placed in each box so that the sum of the numbers in two adjacent boxes is always odd.

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- (c) If you now draw a polygon with 51 sides and on each side you place 50 boxes, taking care that there is a box at each vertex. Can you place a natural number in each box so that the sum of the numbers in two adjacent boxes is always odd? Why?

- 5 In an electronic game of questions and answers, for each correct answer the player adds 5 points on the screen, for each incorrect answer 2 points are subtracted and when the player does not answer, no score is added or subtracted. Each game has 30 questions. Francisco played 5 games and in all of them he obtained the same number of points, greater than zero, but the number of correct answers, errors and unanswered questions in each game was different. Give all the possible scores that Francisco could obtain.