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– level 2

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**1** Veronica, Ana and Gabriela are forming a round and have fun with the following game. One of them chooses a number and says out loud, the one to its left divides it by its largest prime divisor and says the result out loud and so on. The one who says the number out loud 1 wins, at which point the game ends. Ana chose a number greater than 50 and less than 100 and won. Veronica chose the number following the one chosen by Ana and also won. Determine all the numbers that could have been chosen by Ana.

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**2** The owner of a hardware store bought a quantity of screws in closed boxes and sells the screws separately. He never has more than one open box. At the end of the day Monday there are 2208 screws left, at the end of the day Tuesday there are still 1616 screws and at the end of Wednesday there are still 973 screws. To control the employees, every night he writes down the number of screws that are in the only open box. The amount entered on Tuesday is double that of Monday. How many screws are there in each closed box if it is known that they are less than 500?

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**3** It is initially considered a number of three different digits, none of which is equal to zero. Changing instead two of its digits meet a second number less than the first. If the difference between the first and second is a two-digit number and the sum of the first and the second is a palindromic number less than 500, what are the palindromics that can be obtained?

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**4** Consider a pyramid whose base is an equilateral triangle  $BCD$  and whose other faces are triangles isosceles, right at the common vertex  $A$ . An ant leaves the vertex  $B$  arrives at a point  $P$  of the  $CD$  edge, from there goes to a point  $Q$  of the edge  $AC$  and returns to point  $B$ . If the path you made is minimal, how much is the angle  $PQA$ ?

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**5** We have 105 coins, among which we know that there are three fake ones. Authentic coins have all the same weight, which is greater than that of the false ones, which also have the same weight. Determine from can 26 authentic coins be selected by weighing only two in one two pan balance.

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– level 1

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**1** The management of a secret society is made up of 4 people. To admit new partners they use the following criteria: • Only the 4 members of the directory vote, being able to do it in 3 ways: in favor, against or abstaining. • Each aspiring partner must obtain at least 2 votes in favor and none against.

At the last management meeting, 8 requests for admission were examined. Of the total votes

cast, there were 23 votes in favor, 2 votes against and 7 abstaining. What is the highest and what is the lowest value that the number of approved admission requests can have on that occasion?

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- 2** Julia has 289 coins stored in boxes: All the boxes contain the same number of coins (which is greater than 1) and in each box there are coins from the same country, The coins from Bolivia are more than 6% of the total, those from Chile are more than 12% of the total, those of Mexico are more than 24% of the total and those of Peru more than 36% of the total. Can Julia have any coins from Uruguay?
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- 3** Rodolfo and Gabriela have 9 chips numbered from 1 to 9 and they have fun with the following game: They remove the chips one by one and alternately (until they have 3 chips each), with the following rules: • Rodolfo begins the game, choosing a chip and in the following moves he must remove, each time, a chip three units greater than the last chip drawn by Gabriela. • Gabriela, on her turn, chooses a first chip and in the following times she must draw, each time, a chip two units smaller than the last chip that she herself drew. • The game is won by whoever gets the highest number by adding up their three tokens. • If the game cannot be completed, a tie is declared.  
If they play without making mistakes, how should Rodolfo play to be sure he doesn't lose?
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- 4** We have four white equilateral triangles of 3 cm on each side and join them by their sides to obtain a triangular base pyramid. At each edge of the pyramid we mark two red dots that divide it into three equal parts. Number the red dots, so that when you scroll them in the order they were numbered, result a path with the smallest possible perimeter. How much does that path measure?
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- 5** A tortoise walks 60 meters per hour and a lizard walks at 240 meters per hour. There is a rectangle  $ABCD$  where  $AB = 60$  and  $AD = 120$ . Both start from the vertex  $A$  and in the same direction ( $A \rightarrow B \rightarrow D \rightarrow A$ ), crossing the edge of the rectangle. The lizard has the habit of advancing two consecutive sides of the rectangle, turning to go back one, turning to go forward two, turning to go back one and so on. How many times and in what places do the tortoise and the lizard meet when the tortoise completes its third turn?
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