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– level 2

1 How many seven-digit numbers are multiples of 388 and end in 388?

2 In a square $ABCD$ with side k , let P and Q in BC and DC respectively, where $PC = 3PB$ and $QD = 2QC$. Let M be the point of intersection of the lines AQ and PD , determine the area of QMD in function of k

3 There are 10000 equal tiles in the shape of an equilateral triangle. With these little triangles, regular hexagons are formed, without overlaps or gaps. If the regular hexagon that wastes the fewest triangles is formed, how many triangles are left over?

4 In the figures, the vertices are marked with a circle. The segments that join vertices are called paths. Non-negative integers are distributed to the vertices and, to the paths, the differences between the numbers at their ends.

<https://cdn.artofproblemsolving.com/attachments/d/6/e6fce93719a5b35dbf34d58652b01a8631de5>
gif

We will say that a distribution of numbers is *graceful* if all the numbers from 1 to n appear in the paths, where n is the number of paths.

The following is an example of graceful distribution:

<https://cdn.artofproblemsolving.com/attachments/1/1/a8c2b4fde673ca902b655804c4f5321f9666e>
gif

Give -if possible- a graceful distribution for the following figures. If you can't do it, show why.

5 What are the possible areas of a hexagon with all angles equal and sides 1, 2, 3, 4, 5, and 6, in some order?

– level 1

1 On a square board with 9 squares (three by three), nine elements of the set $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ must be placed, different from each other, so that each one is in a box and the following conditions are met: • The sums of the numbers in the second and third rows are, respectively, double and triple the sum of the numbers in the first row. • The sum of the numbers in the second and third columns are, respectively, double and triple the sum of the numbers in the first column. Show all the possible ways to place elements of S on the board, fulfilling the indicated conditions.

- 2 In the rectangle $ABCD$, M , N , P and Q are the midpoints of the sides. If the area of the shaded triangle is 1, calculate the area of the rectangle $ABCD$.
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- 3 On an 8×8 board, 10 checkers have been placed, each occupying a square. On each square without a token, a number between 0 and 8 is written, which is equal to the number of tokens placed on its neighboring squares. Neighboring cells are those that have a side or a vertex in common. Give a distribution of the tiles that makes the sum of the numbers written on the board the greatest possible.
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- 4 Joaquín and his brother Andrés go to class every day on the 62 bus. Joaquín always pays for the tickets. Each ticket has a 5-digit number printed on it. One day, Joaquín observes that the numbers on his tickets - his and his brother's - as well as being consecutive, are such that the sum of the ten digits is precisely 62. Andrés asks him if the sum of the digits of any of the tickets is 35 and, knowing the answer, he can directly say the number of each ticket. What were those numbers?
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- 5 When Pablo turns 15, he throws a party inviting 43 friends. He presents them with a cake in the form of a regular 15-sided polygon and on it 15 candles. The candles are arranged so that between candles and vertices there are never three aligned (any three candles are not aligned, nor are any two candles with a vertex of the polygon, nor are any two vertices of the polygon with a candle). Then Pablo divides the cake into triangular pieces, by means of cuts that join candles to each other or candles and vertices, but also do not intersect with others already made. Why, by doing this, Paul was able to distribute a piece to each of his guests but he was left without eating?
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