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– level 2

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**1** A three-digit natural number is called *tricubic* if it is equal to the sum of the cubes of its digits. Find all pairs of consecutive numbers such that both are tricubic.

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**2** In a unit circle where  $O$  is your circumcenter, let  $A$  and  $B$  points in the circle with  $\angle BOA = 90$ . In the arc  $AB$ (minor arc) we have the points  $P$  and  $Q$  such that  $PQ$  is parallel to  $AB$ . Let  $X$  and  $Y$  be the points of intersections of the line  $PQ$  with  $OA$  and  $OB$  respectively. Find the value of  $PX^2 + PY^2$

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**3** The first row of this table is filled with the numbers 1 through 10, in that order. The second row is filled with the numbers from 1 to 10, in any order. In each box of the third row the sum of the two numbers written above is written. Is there a way to fill in the second row so that the ones digits of the numbers in the third row are all different?

<https://cdn.artofproblemsolving.com/attachments/8/5/41117d105cc880bf452fa46132c20f2167aa5.png>

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**4** Let  $ABC$  be an equilateral triangle.  $M$  is the midpoint of segment  $AB$  and  $N$  is the midpoint of segment  $BC$ . Let  $P$  be the point outside  $ABC$  such that the triangle  $ACP$  is isosceles and right in  $P$ .  $PM$  and  $AN$  are cut in  $I$ . Prove that  $CI$  is the bisector of the angle  $MCA$ .

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**5** There are 12 points that are vertices of a regular polygon with 12 sides. Rafael must draw segments that have their two ends at two of the points drawn. He is allowed to have each point be an endpoint of more than one segment and for the segments to intersect, but he is prohibited from drawing three segments that are the three sides of a triangle in which each vertex is one of the 12 starting points. Find the maximum number of segments Rafael can draw and justify why he cannot draw a greater number of segments.

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– level 1

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**1** Two integers between 1 and 100 inclusive are chosen such that their difference is 7 and their product is a multiple of 5. In how many ways can this choice be made?

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**2** In a parallelogram  $ABCD$ ,  $BD$  is the largest diagonal. By matching  $B$  with  $D$  by a bend, a regular pentagon is formed. Calculate the measures of the angles formed by the diagonal  $BD$  with each of the sides of the parallelogram.

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**3** On each step of a ladder with 10 steps there is a frog. Each of them can, in one jump, be placed on another step, but when it does, at the same time, another frog will jump the same number of steps in the opposite direction: one goes up and another goes down. Will the frogs manage to get all together on the same step?

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**4** Ten square cardboards of 3 centimeters on a side are cut by a line, as indicated in the figure. After the cuts, there are 20 pieces: 10 triangles and 10 trapezoids. Assemble a square that uses all 20 pieces without overlaps or gaps.

<https://cdn.artofproblemsolving.com/attachments/7/9/ec2242cca617305b02eef7a5409e6a6b482d6.gif>

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**5** Ana, Beatriz, Carlos, Diego and Emilia play a chess tournament. Each player faces each of the other four only once. Each player gets 2 points if he wins the match, 1 point if he draws and 0 point if he loses. At the end of the tournament, it turns out that the scores of the 5 players are all different. Find the maximum number of ties there could be in the tournament and justify why there could not be a higher number of ties.

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