VI - Rioplatense Mathematical Olympiad, Level 3 1997

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–	Day 1
1	Find all positive integers n with the following property: there exists a polynomial $P_n(x)$ of degree n, with integer coefficients, such that $P_n(0) = 0$ and $P_n(x) = n$ for n distinct integer solutions.
2	Consider a prism, not necessarily right, whose base is a rhombus $ABCD$ with side $AB = 5$ and diagonal $AC = 8$. A sphere of radius r is tangent to the plane $ABCD$ at C and tangent to the edges AA_1 , BB_1 and DD_1 of the prism. Calculate r .
3	Prove that there are infinitely many positive integers n such that the number of positive divisors in $2^n - 1$ is greater than n.
–	Day 2
4	Circles c_1 and c_2 are tangent internally to circle c at points A and B , respectively, as seen in the figure. The inner tangent common to c_1 and c_2 touches these circles in P and Q , respectively. Show that the AP and BQ lines intersect the circle c at diametrically opposite points. `https://cdn.artofproblemsolving.com/attachments/0/a/9490a4d7ba2038e490a858b14ba21d07377c5.gif`
5	Let $x_1, x_2, ..., x_n$ be non-negative numbers $n \geq 3$ such that $x_1 + x_2 + ... + x_n = 1$. Determine the maximum possible value of the expression $x_1x_2 + x_2x_3 + ... + x_{n-1}x_n$.
6	Let N be the set of positive integers. Determine if there is a function $f : N \to N$ such that $f(f(n)) = 2n$, for all n belongs to N.