II - Rioplatense Mathematical Olympiad, Level 3 1992

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– Day 1

1 Let $f : Z \to N - \{0\}$ such that:

$f(x + y)f(x - y) = (f(x)f(y))^2$ and $f(1) \neq 1$.

Provethat $\log_{f(1)} f(z)$ is a perfect square for every integer z.

2 Determine the integers $0 \leq a \leq b \leq c \leq d$ such that:

$$2^n = a^2 + b^2 + c^2 + d^2.$$

3 Let D be the center of the circumcircle of the acute triangle ABC. If the circumcircle of triangle ADB intersects AC (or its extension) at M and also BC (or its extension) at N, show that the radii of the circumcircles of $\triangle ADB$ and $\triangle MNC$ are equal.

– Day 2

4 On the planet Mars there are 100 states that are in dispute. To achieve a peace situation, blocs must be formed that meet the following two conditions:
(1) Each block must have at most 50 states.
(2) Every pair of states must be together in at least one block.
Find the minimum number of blocks that must be formed.

5 Let ABC be an acute triangle.
Find the locus of the centers of the rectangles which have their vertices on the sides of ABC.

6 Definition: A natural number is *abundant* if the sum of its positive divisors is greater than its double.

Find an odd abundant number and prove that there are infinitely many odd abundant numbers.