III - Rioplatense Mathematical Olympiad, Level 3 1993

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– Day 1

1 Find all functions f defined on the integers greater than or equal to 1 that satisfy:
(a) for all $n, f(n)$ is a positive integer.
(b) $f(n+m) = f(n)f(m)$ for all m and n.
(c) There exists n_0 such that $f(f(n_0)) = [f(n_0)]^2$.

2 An integer is written in each cell of a board of$N$ rows and $N+1$ columns. Prove that some columns (possibly none) can be deleted so that in each row the sum of the numbers left uncrossed out is even.

3 Given three points A, B and C (not collinear) construct the equilateral triangle of greater perimeter such that each of its sides passes through one of the given points.

– Day 2

4 x and y are real numbers such that $6-x$, $3+y^2$, $11+x$, $14-y^2$ are greater than zero.
Find the maximum of the function

$$f(x,y) = \sqrt{(6-x)(3+y^2)} + \sqrt{(11+x)(14-y^2)}.$$

5 Prove that for every integer $k \geq 2$ there are k different natural numbers n_1, n_2, ... , n_k such that:

$$\frac{1}{n_1} + \frac{1}{n_2} + ... + \frac{1}{n_k} = \frac{3}{17}$$

6 Let $ABCDE$ be pentagon such that $AE = ED$ and $BC = CD$. It is known that $\angle BAE + \angle EDC + \angle CBA = 360^o$ and that P is the midpoint of AB. Show that the triangle ECP is right.