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by parmenides51, mathisreal

– level 2

1 Determine the smallest positive integer that has all its digits equal to 4, and is a multiple of 169.

2 Let $ABCD$ be a rectangle and the circle of center D and radius DA, which cuts the extension of the side AD at point P. Line PC cuts the circle at point Q and the extension of the side AB at point R. Show that $QB = BR$.

3 Find the minimum $k > 2$ for which there are k consecutive integers such that the sum of their squares is a square.

4 Let n be a integer $1 < n < 2010$, where we have a polygon with 2010 sides and n coins, we have to paint the vertices of this polygon with n colors and we've to put the n coins in n vertices of the polygon. In each second the coins will go to the neighbour vertex, going in the clockwise. Determine the values of n such that is possible paint and choose the initial position of the coins where in each second the n coins are in vertices of distinct colors

5 You have the following pieces: one 4×1 rectangle, two 3×1 rectangles, three 2×1 rectangles, and four 1×1 squares. Ariel and Bernardo play the following game on a board of $n\times n$, where n is a number that Ariel chooses. In each move, Bernardo receives a piece R from Ariel. Next, Bernardo analyzes if he can place R on the board so that it has no points in common with any of the previously placed pieces (not even a common vertex). If there is such a location for R, Bernardo must choose one of them and place R. The game stops if it is impossible to place R in the way explained, and Bernardo wins. Ariel wins only if all 10 pieces have been placed on the board.
a) Suppose Ariel gives Bernardo the pieces in decreasing order of size. What is the smallest n that guarantees Ariel victory?
b) For the n found in a), if Bernardo receives the pieces in increasing order of size, is Ariel guaranteed victory?

Note: Each piece must cover exactly a number of unit squares on the board equal to its own size. The sides of the pieces can coincide with parts of the edge of the board.

– level 1

1 A closed container in the shape of a rectangular parallelepiped contains 1 liter of water. If the container rests horizontally on three different sides, the water level is 2 cm, 4 cm and 5 cm. Calculate the volume of the parallelepiped.

2 In stage 0 the numbers are written: $1, 1$.
In stage 1 the sum of the numbers is inserted: $1, 2, 1$.
In stage 2, between each pair of numbers from the previous stage, the sum of them is inserted: $1, 3, 2, 3, 1$.
One more stage: $1, 4, 3, 5, 2, 5, 3, 4, 1$.
How many numbers are there in stage 10?
What is the sum of all the numbers in stage 10?

3 Is it possible to color positive integers with three colors so that whenever two numbers with different colors are added, the result of their addition is the third color? (All three colors must be used.) If the answer is yes, indicate a possible coloration; if not, explain why.

4 Find all natural numbers of 90 digits that are multiples of 13 and have the first 43 digits equal to each other and nonzero, the last 43 digits equal to each other, and the middle 4 digits are $2, 0, 1, 0$, in that order.

5 In a 2×7 board gridded in 1×1 squares, the 24 points that are vertices of the squares are considered.
`https://cdn.artofproblemsolving.com/attachments/9/e/841f11ef9d6fc27cdbe7c91bab6d52d12180e gif`
Juan and Matías play on this board. Juan paints red the same number of points on each of the three horizontal lines. If Matthias can choose three red dots that are vertices of an acute triangle, Matthias wins the game. What is the maximum number of dots Juan can color in to make sure Matías doesn't win? (For the number found, give an example of coloring that prevents Matías from winning and justify why if the number is greater, Matías can always win.)