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– level 2

1 Initially, the number 1 is written on the blackboard. At each step, the number on the blackboard is erased and another is written, which is obtained by applying any of the following operations:

Operation A: Multiply the number on the board with $\frac{1}{2}$.
Operation B: Subtract the number on the board from 1.

For example, if the number $\frac{3}{8}$ is on the board, it can be replaced by $\frac{1}{2}\frac{3}{8} = \frac{3}{16}$ or by $1 - \frac{3}{8} = \frac{5}{8}$.

Give a sequence of steps after which the number on the board is $\frac{2009}{2^{20009}}$.

2 Let $ABCD$ be a convex quadrilateral such that the triangle ABD is equilateral and the triangle BCD is isosceles, with $\angle C = 90^o$. If E is the midpoint of the side AD, determine the measure of the angle $\angle CED$.

3 In the following sum: $1 + 2 + 3 + 4 + 5 + 6$, if we remove the first two "+" signs, we obtain the new sum $123 + 4 + 5 + 6 = 138$. By removing three "+" signs, we can obtain $1 + 23 + 456 = 480$. Let us now consider the sum $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13$, in which some "+" signs are to be removed. What are the three smallest multiples of 100 that we can get in this way?

4 Each square of a 5×5 board is painted red or blue, in such a way that the following condition is fulfilled: "For any two rows and two columns, of the 4 squares that are in their intersections, there are 4, 2 or 0 painted red." How many ways can the board be painted?

5 A game of solitaire strats of with 25 cards. Some are facing up and sum are facing down.
In each move a card that's facing up should me choosen, taken away, and turning over the cards next to it (if there are cards next to it). The game is won when you have accomplished to take all the 25 cards from the table.
If you initially start with n cards facing up, find all the values of n such that the game can be won. Explain how to win the game, independently from the initial placement of the cards facing up, justify your answer for why it is impossible to win with other values of n.
Two cards are neighboring when one is immediately next to the other, to the left or right.

Example: The card marked A has two neighboring cards and the one marked with only a B has only one neighboring card. After taking a card there is a hole left, such that the card marked C has only one neighboring card, and the one marked D does'nt have any.

– level 1

1 Each two-digit natural number is *assigned* a digit as follows: Its digits are multiplied. If the result is a digit, this is the assigned digit. If the result is a two-digit number, these two figures are multiplied, and if the result is a digit, this is the assigned digit. Otherwise, the operation is repeated. For example, the digit assigned to 32 is 6 since $3\times = 6$; the digit assigned to 93 is 4 since $9 \times 3 = 27$, $2 \times 7 = 14$, $1 \times 4 = 4$. Find all the two-digit numbers that are assigned 8.

2 Find prime numbers p, q, r such that $p + q^2 + r^3 = 200$. Give all the possibilities.

Remember that the number 1 is not prime.

3 There are 26 cards and each one has a number written on it. There are two with 1, two with 2, two with 3, and so on up to two with 12 and two with 13. You have to distribute the 26 cards in piles so that the following two conditions are met: • If two cards have the same number they are in the same pile. • No pile contains a card whose number is equal to the sum of the numbers of two cards in that same pile.
Determine what is the minimum number of stacks to make. Give an example with the distribution of the cards for that number of stacks and justify why it is impossible to have fewer stacks.

Clarification: Two squares are *neighbors* if they have a common side.

4 Three circumferences are tangent to each other, as shown in the figure. The region of the outer circle that is not covered by the two inner circles has an area equal to 2 p. Determine the length of the PQ segment .
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png

5 An ant walks along the lines of a grid made up of 55 horizontal lines and 45 vertical lines. You want to paint some sections of lines so that the ant can go from any intersection to any other intersection, walking exclusively along painted sections. If the distance between consecutive lines is 10 cm, what is the least possible number of centimeters that should be painted? What is the higher value?