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– level 2

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1 In a blackboard, it's written the following expression

$1-2-2^2-2^3-2^4-2^5-2^6-2^7-2^8-2^9-2^{10}$

We put parenthesis by different ways and then we calculate the result. For example:

$1-2-\left(2^2-2^3\right)-2^4-\left(2^5-2^6-2^7\right)-2^8-\left(2^9-2^{10}\right)=403$ and

$1-\left(2-2^2\left(-2^3-2^4\right)-\left(2^5-2^6-2^7\right)\right)-\left(2^8-2^9\right)-2^{10}=-933$

How many different results can we obtain?

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2 Let $ABCD$ be a rectangle and P be a point on the sideAD such that $\angle BPC=90^o$. The perpendicular from A on BP cuts BP at M and the perpendicular from D on CP cuts CP in N. Show that the center of the rectangle lies in the MN segment.

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3 In numbers $1010...101$ Ones and zeros alternate, if there are n ones, there are $n-1$ zeros ($n\geq 2$).Determine the values of n for which the number $1010...101$, which has n ones, is prime.

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4 In the plane we have 16 lines(not parallel and not concurrents), we have 120 point(s) of intersections of this lines.
Sebastian has to paint this 120 points such that in each line all the painted points are with colour differents, find the minimum(quantity) of colour(s) that Sebastian needs to paint this points.
If we have have 15 lines(in this situation we have 105 points), what's the minimum(quantity) of colour(s)?

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5 Matthias covered a 7×7 square board, divided into 1×1 squares, with pieces of the following three types without gaps or overlaps, and without going off the board.
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gif
Each type 1 piece covers exactly 3 squares and each type 2 or type 3 piece covers exactly 4 squares.
Determine the number of pieces of type 1 that Matías could have used.
(Pieces can be rotated and flipped.)

– level 1

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1 How many different numbers with 6 digits and multiples of 45 can be written by adding one digit to the left and one to the right of 2008?

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2 In the Olympic school the exams are graded with whole numbers, the lowest possible grade is 0, and the highest is 10. In the arithmetic class the teacher takes two exams. This year he has 15 students. When one of his students gets less than 3 on the first exam and more than 7 on the second exam, he calls him an overachieving student. The teacher, at the end of correcting the exams, averaged the 30 grades and obtained 8. What is the largest number of students who passed this class could have had?

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3 On a blackboard are written all the integers from 1 to 2008 inclusive. Two numbers are deleted and their difference is written. For example, if you erase 5 and 241, you write 236. This continues, erasing two numbers and writing their difference, until only one number remains. Determine if the number left at the end can be 2008. What about 2007? In each case, if the answer is affirmative, indicate a sequence with that final number, and if it is negative, explain why.

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4 Let ABF be a right-angled triangle with $\angle AFB = 90$, a square $ABCD$ is externally to the triangle. If $FA = 6$, $FB = 8$ and E is the circumcenter of the square $ABCD$, determine the value of EF

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5 On a $16x16$ board, 25 coins are placed, as in the figure. It is allowed to select 8 rows and 8 columns and remove from the board all the coins that are in those 16 lines. Determine if it is possible to remove all coins from the board.
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`gif`
If the answer is yes, indicate the 8 rows and 8 columns selected, and if no, explain why.
