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– level 2

1 Determine the largest natural number that has all its digits different and is a multiple of 5, 8 and 11.

2 Let $n > 2$ be an even integer. In the squares of a board of $n \times n$, pieces must be placed so that in each column the number of chips is even and different from zero, and in each row the number of chips is odd. Determine the fewest number of checkers to place on the board to satisfy this rule. To show a configuration with that number of tokens and explain why with fewer tokens the rule.

3 Eight children, all of different heights, must form an orderly line from smallest to largest. We will say that the row has exactly one error if there is a child that is immediately behind another taller than it, and everyone else (except the first in line) is immediately behind a shorter one. of how many ways the eight children can line up with exactly one mistake?

4 Alex and Bruno play the following game: each one, in your turn, the player writes, exactly one digit, in the right of the last number written. The game finishes if we have a number with 6 digits(distincts) and Alex starts the game. Bruno wins if the number with 6 digits is a prime number, otherwise Alex wins.
Which player has the winning strategy?

5 In the triangle ABC we have $\angle A = 2\angle C$ and $2\angle B = \angle A + \angle C$. The angle bisector of $\angle C$ intersects the segment AB in E , let F be the midpoint of AE , let AD be the altitude of the triangle ABC . The perpendicular bisector of DF intersects AC in M .
Prove that $AM = CM$.

– level 1

1 In a year that has 53 Saturdays, what day of the week is May 12? Give all chances.

2 Let $X = a1b9$ and $Yab = 51ab$ be two positive integers where a and b are digits. X is known to be multiple of a positive two-digit number n and Y is the next multiple of that number n . Find the number n and the digits a and b . Justify why there are no other possibilities.

3 Jorge chooses 6 different positive integers and writes one on each face of a cube. He threw his bucket three times.
The first time his cube showed the number 5 facing up and also the sum of the numbers on

the faces sides was 20. The second time his cube showed the number 7 facing up and also the sum of the numbers on the faces sides was 17. The third time his cube showed the number 4 up, plus all the numbers on the side faces. They turned out to be primes. What are the numbers that Jorge chose and how did he distribute them on the faces of the cube? Analyze all odds.

Remember that 1 is not prime.

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- 4** A 7×7 board has a lamp on each of its 49 squares, which can be on or off. The allowed operation is to choose 3 consecutive cells of a row or a column that have two lamps neighboring each other on and the other off, and change the state of all three. Namely <https://cdn.artofproblemsolving.com/attachments/e/b/28737b19c940ff5e1c98d05533c77069e9901.png>
Give a configuration of exactly 8 lit lamps located in the first 4 rows of the board such that, through a succession of permitted operations, a single lamp is lit on the board and that it is located in the last row. Show the sequence of operations used to achieve the goal.
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- 5** You have a paper pentagon, $ABCDE$, such that $AB = BC = 3$ cm, $CD = DE = 5$ cm, $EA = 4$ cm, $\angle ABC = 100^\circ$, $\angle CDE = 80^\circ$. You have to divide the pentagon into four triangles, by three straight cuts, so that with the four triangles assemble a rectangle, without gaps or overlays. (The triangles can be rotated and / or turned around.)
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