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by parmenides51

–   level 2

1   Determine all pairs of natural numbers a and b such that $\frac{a+1}{b}$ and $\frac{b+1}{a}$ they are natural numbers.

2   Several prime numbers (some repeated) are written on the board. Mauro added the numbers on the board and Fernando multiplied the numbers on the board. The result obtained by Fernando is equal to 40 times the result obtained by Mauro. Determine what the numbers on the board can be. Give all chances.

3   Write a positive integer in each box so that:
All six numbers are different.
The sum of the six numbers is 100.
If each number is multiplied by its neighbor (in a clockwise direction) and the six results of those six multiplications are added, the smallest possible value is obtained.
Explain why a lower value cannot be obtained.
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gif

4   Let $ABCD$ be a trapezoid of bases AB and CD . Let O be the intersection point of the diagonals AC and BD. If the area of the triangle ABC is 150 and the area of the triangle ACD is 120, calculate the area of the triangle BCO.

5   With 28 points, a "triangular grid" of equal sides is formed, as shown in the figure.
One operation consists of choosing three points that are the vertices of an equilateral triangle and removing these three points from the grid. If after performing several of these operations there is only one point left, in what positions can that point remain?
Give all the possibilities and indicate in each case the operations carried out.
Justify why the remaining point cannot be in another position.
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–   level 1

1   A digital calendar displays the date: day, month, and year, with 2 digits for the day, 2 digits for the month, and 2 digits for the year. For example, $01 - 01 - 01$ is January 1, 2001 and $05 - 25 - 23$ is May $25, 2023$. In front of the calendar is a mirror. The digits of the calendar are as in the figure
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`gif`

If $0, 1, 2, 5$, and 8 are reflected, respectively, in $0, 1, 5, 2$, and 8, and the other digits lose meaning when reflected, determine how many days of the century, when reflected in the mirror, also correspond to a date.

2   A rectangle of paper of 3 cm by 9 cm is folded along a straight line, making two opposite vertices coincide. In this way a pentagon is formed. Calculate your area.

3   There are 20 collinear points, separated by the same distance:

. . . .

Miguel has to paint three or more of these points red, in such a way that the red points are separated by the same distance and it is impossible to paint exactly one more point red without violating the previous condition. Determine in how many ways Miguel can do his homework.

4   With 150 white cubes of $1 \times 1 \times 1$ a prism of $6 \times 5 \times 5$ is assembled, its six faces are painted blue and then the prism is disassembled. Lucrecia must build a new prism, without holes, exclusively using cubes that have at least one blue face and so that the faces of Lucrecia's prism are all completely blue.
Give the dimensions of the prism with the largest volume that Lucrecia can assemble.

5   In some squares of a 10×10 board, a piece is placed in such a way that the following property is satisfied: For each square that has a piece, the number of pieces placed in the same row must be greater than or equal to the number of pieces placed in the same column. How many tiles can there be on the board? Give all chances.